IM2 Problem Set 6.8 - Exponential Functions

BIG PICTURE of this UNIT:	● How can we analyze growth or decay patterns in data sets & contextual problems? ● How can we algebraically & graphically summarize growth or decay patterns? ● How can we compare & contrast linear and exponential models? ● How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CI)** Evaluate each of the following expressions:

   a. $\sqrt[9]{512} + \sqrt[3]{27^2} - \sqrt[3]{-216}^{\,5}$ b. $\sqrt[5]{\frac{-32}{243}} \times \sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$

2. **(CI)** Given the function $g(x) = 3(2)^{x-2} + 2$:

   a. Evaluate $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, $g(2)$, $g(3)$
   b. determine the x- and y-intercept(s) - if they exist
   c. determine the equation of the asymptote of $g(x)$
   d. sketch $g(x)$, labelling the data points and intercept(s) and the asymptote.

3. **(CA)** Find the logbase command on your TI-84 (use alpha window option 5) and evaluate the following:

   a. (i) $log_2(8) = ?$ (ii) $log_2(16) = ?$ (iii) $log_2(128) = ?$ (iv) $log_2(4096) = ?$
   b. (i) $log_3(27) = ?$ (ii) $log_3(81) = ?$ (iii) $log_3\left(\frac{1}{9}\right) = ?$ (iv) $log_3\left(\frac{1}{243}\right) = ?$
   c. (i) $log_{0.5}(2) = ?$ (ii) $log_{0.5}\left(\frac{1}{16}\right) = ?$ (iii) $log_{0.5}\left(\frac{1}{128}\right) = ?$ (iv) $log_{0.5}(32) = ?$
   d. What seems to be the point of this log function?

Part 2 - Skills/Concepts Application Problems

4. **(CA)** The population of HS students at CAC can be modeled with an exponential function. There were 370 students present in 2011 and 315 students in 2014.

   a. What is the annual rate of decrease in the student population of high school students?
   b. When will the student population be predicted to be 200? What assumptions are you making?

5. **(CA)** Use the logbase command on your calculators to solve the following exponential equations:

   a. $4^{1-2x} = 24$ b. $2^{3-2x} = 50$ c. $2^{3-2x} = 3^x$ d. $5^{1-2x} = 2^x$

6. **(CA)** Solve for *t* in each of the following questions:

   a. You buy a new computer for \$2100. The computer decreases in value by 50% annually. When will the computer be worth \$600?
   b. The population of HS students at CAC since the year 2000 can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. How long would it take for the student population to decline from 350 students to 250 students?
   c. The value of land in New Cairo grows exponentially. Today 10 hectares of land cost 2.5 million LE and the value of the land is increasing at an annual rate of 17.5%. How long will it take for the land value to be 4.0 million LE?

7. **(CA)** The value of land in New Cairo grows exponentially. Five years ago, 10 hectares of land cost 0.75 million LE and today, the same 10 hectares cost 2.5 million LE.

   a. Determine the annual rate of increase of the land.
   b. When would you predict that the cost of 10 hectares of land exceeds 7.5 million LE

8. **(CA)** Examining Changes in the Compounding Conditions. When my oldest son, Alexander, was born, I invested \$5,000 in an education fund for him. The education fund is earning 8% compound interest every year. You will develop an answer to my questions:

   a. How much this investment is worth when Alexander starts university at the age of 19?

When interest is "paid" to the investor, it DOES NOT HAVE TO BE ANNUALLY!!!. What if an investor (like me) wants the interest paid MORE FREQUENTLY? How does this change the value of an investment?? How does it change the formula that I can use to predict future values?
Let's reconsider my first example: When my oldest son, Alexander, was born, my wife and I invested \$5,000 in an education fund for him. The education fund is earning 8% interest every year until Alexander is 19 ⇒ Now I will have 4 investment options that you will investigate to determine the value of these investments (wherein I simply change the frequency by which the interest is paid to me)

OPTION A ⇒ 8%/a compounded semi-annually
OPTION B ⇒ 8%/a compounded quarterly
OPTION C ⇒ 8%/a compounded monthly
OPTION D ⇒ 8%/a compounded daily

   b. Summary ⇒ Does the value of my investment for Alex change in value given the different compounding conditions? Any ideas as to WHY/WHY NOT?

   c. Does the formula I use to predict future values change given the different compounding conditions?

9. **(CA)** For each situation, determine: (i) the amount (value of the investment) (ii) the interest earned

   a. \$4000 borrowed for 4 years at 3%/a, compounded annually
   b. \$7500 invested for 6 years at 6%/a, compounded monthly
   c. \$15 000 borrowed for 5 years at 2.4%/a, compounded quarterly
   d. \$28 200 invested for 10 years at 5.5%/a, compounded semi-annually
   e. \$850 financed for 1 year at 3.65%/a, compounded daily
   f. \$2225 invested for 47 weeks at 5.2%/a, compounded weekly

10. **(CA)** Sally invests some money at 6%/a compounded annually. After 5 years, she takes the principal and the interest earned and reinvests it all at 7.2%/a compounded quarterly for 6 more years. At the end of this time, her investment is worth 14, 784.56. How much did Sally invest initially?

11. **(CA)** On the day Rachel was born, her grandparents deposited \$5000 into a savings account that earns 4.8%/a compounded monthly. They deposit the same amount on her 5th, 10th and 15th birthdays. Determine the balance in the account on Rachel's 18th birthday.