IM2 Problem Set 6.6 - Exponential Functions

BIG PICTURE of this UNIT:	- How can we analyze growth or decay patterns in data sets & contextual problems? - How can we algebraically & graphically summarize growth or decay patterns? - How can we compare & contrast linear and exponential models? - How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CI)** Mr D makes the following observation about exponents and exponent rules:

   (i) $4^1 \times 4^1 = 4^2$ (ii) $4^2 \times 4^2 = 4^4$ (iii) $4^3 \times 4^3 = 4^6$

   So he wonders what would happen in the following situation: $4^{\#} \times 4^{\#} = 4^1$

   a. What value does # have?
   b. What does $4^{\#}$ equal?

2. **(CI)** Mr D makes the following observation about exponents and exponent rules:

   (i) $8^1 \times 8^1 \times 8^1 = 8^3$ (ii) $8^2 \times 8^2 \times 8^2 = 8^6$ (iii) $8^3 \times 8^3 \times 8^3 = 8^9$

   So he wonders what would happen in the following situation: $8^{\#} \times 8^{\#} \times 8^{\#} = 8^1$

   a. What value does # have?
   b. What does $8^{\#}$ equal?

3. **(CI)** Simplify the following expressions using the appropriate exponent laws and operations.

   a. $\dfrac{(6x^3y^{-4})^{-2}}{(3x^2y^5)^{-3}}$ b. $\dfrac{(8x^3y^{-4})^{-2}}{(-4x^{-1}y^2)^{-3} \cdot (2x^5y^{-3})^{-2}}$ c. $\dfrac{x^{-1} + y^{-1}}{(xy)^{-2}}$

4. **(CA)** A tool & die business purchased a piece of equipment of $250,000. The value of the equipment depreciates at a rate of 12% each year.

   a. Write an exponential decay model for the value of equipment.
   b. What is the value of equipment after 5 years?
   c. Estimate when the equipment will have a value of $70,000
   d. What is the monthly rate of depreciation

5. **(CI)** Evaluate the following expressions:

a. (i) $8^{-\frac{2}{3}}$ (ii) $25^{-\frac{3}{2}}$ (iii) $16^{-\frac{5}{4}}$ (iv) $81^{-\frac{3}{4}}$

b. (i) $3^{\frac{4}{3}} \times 3^{\frac{5}{3}}$ (ii) $(7^3)^{\frac{2}{3}}$ (iii) $8^{-\frac{5}{3}} \times 8^{\frac{6}{3}}$

Part 2 - Skills/Concepts Application Problems

6. **(CA)** The half-life of a medication is the amount of time for half of the drug to be eliminated from the body. The half-life of *Advil* or ibuprofen is represented by the equation $R(t) = M\left(\frac{1}{2}\right)^{\frac{t}{2}}$, where R is the amount of Advil remaining in the body, M is the initial dosage, and t is time in hours since a dose was taken.

   a. How much Advil is left after 2 hours? Hence, what is the half life of Advil?
   b. A 200 milligram dosage of Advil is taken at 11:00 am. How many milligrams of the medication will remain in the body at 5:00 pm?
   c. Mr R is taking an Advil every 12 hours and he takes a 200 milligram dosage of Advil at 11:00 am, how many milligrams of the medication will remain in the body 12 hours later.
   d. He then takes another dose at 11:00 pm, how many milligrams of the medication will be in his body at that time?

7. **(CI)** Given the function $g(x) = 16 - 2^{x+2}$:

   a. evaluate $g(-3)$, $g(-2)$, $g(-1)$, $g(0)$, $g(1)$, $g(2)$
   b. determine the x- and y-intercept(s), if they exist
   c. determine the equation of the asymptote of $g(x)$
   d. sketch $g(x)$, labelling the data points and intercept(s) and the asymptote.

8. **(CA)** In 1990 the cost of attending University of Math was $15000. During the next 25 years, the cost has increased by an average of 5.2% per year.

   a. Write a model giving the cost, $C(t)$, at University of Math t years after 1990.
   b. In what year did the tuition exceed $30,000?
   c. Estimate the tuition in 2020 - the year you will attend this college!
   d. Mr S has set up a college fund for his son Ian. This fund started in 2000 with an initial investment of $30,000 and has grown at 5.8% every year. If Ian attends a three year program at U of Math, can this college fund pay for these costs? Show your work/reasoning.

9. **(CI)** Solve the following equations and verify your solutions.

a. (i) $2^{3-x} = 2^4$ (ii) $2^{x-3} = 2^{3x+1}$ (iii) $2^{2x+3} = 16$
b. (i) $2^{1-2x} = 8$ (ii) $3^{x+2} = \frac{1}{9}$ (iii) $8^x = 16^{x-1}$

10. **(CA)** Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of $f(x) = ab^x$.

x	2	3	4	5	6	7	8
$f(x)$	28.1	21.2	16.1	11.9	9.1	6.6	5.0

a. Determine the "percent change" between each pair of terms:
$\%\text{ change} = \frac{y(3)-y(2)}{y(2)}$; $\%\text{ change} = \frac{y(4)-y(3)}{y(3)}$; $\%\text{ change} = \frac{y(5)-y(4)}{y(4)}$; etc ….
b. This creates an equation in the form of $y = a(1+r)^x$. Use a data point to find the value of a and now, what equation models this data set?
c. Secondly, now determine the "common ratio" between each pair of terms (you do this by dividing the successive y terms ==> ratio = $\frac{y(3)}{y(2)}$; ratio = $\frac{y(4)}{y(3)}$; r = $\frac{y(5)}{y(4)}$; etc …..
d. Finally, enter the data set into the lists L_1 and L2 in your calculator and use the TI-84 to determine the equation of the exponential function that models this data set.
e. Finally, what is the equation for this data set?

11. **(CI)** For each of the functions, state their domain and range and the equation of the asymptote. Sketch each function, labeling the y-intercept, the x-intercept (if they exist) t, the asymptote.

a. First, graph the function $g(x) = 2^x$.
b. Then, graph each of the following functions (that are somehow related to $g(x)$)

i. $g(x) = 2^x - 3$
ii. $g(x) = 2^{x-3}$
iii. $g(x) = -2^x + 3$
iv. $g(x) = 2^{-x} + 4$