IM2 Problem Set 6.4 - Exponential Functions

BIG PICTURE of this UNIT:	● How can we analyze growth or decay patterns in data sets & contextual problems? ● How can we algebraically & graphically summarize growth or decay patterns? ● How can we compare & contrast linear and exponential models? ● How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CI)** Which of the functions listed below are "growth" functions and which are "decay" functions? For each function, determine the **growth/decay factor** as well as the **growth/decay rate**.

   a. $y = 5(2)^x$ b. $y = 100(0.5)^x$ c. $y = 80(1.3)^x$ d. $y = 20(0.8)^x$

2. **(CA)** Use your calculator to work through the following questions:

   a. Find the value of: (i) $9^{\frac{1}{2}}$ (ii) $16^{\frac{1}{2}}$ (iii) $36^{\frac{1}{2}}$ (iv) $225^{\frac{1}{2}}$ (v) $900^{\frac{1}{2}}$
   b. Explain your values and explain what the exponent of ½ means
   c. Find the value of: (i) $8^{\frac{1}{3}}$ (ii) $27^{\frac{1}{3}}$ (iii) $125^{\frac{1}{3}}$ (iv) $343^{\frac{1}{3}}$ (v) $1000^{\frac{1}{3}}$
   d. Explain your values and explain what the exponent of ⅓ means
   e. What would the exponents ¼ and ⅕ then mean?

3. **(CI/CA)** Solve the following equations using "inverse operations":

   a. $3x + 3 = 11$ b. $\frac{x}{3} + 3 = 11$ c. $x^2 + 3 = 11$ d. $x^3 + 3 = 11$ e. $3^x + 3 = 11$

Part 2 - Skills/Concepts Application Problems

4. **(CA)** Use your calculator to evaluate the following expressions and then summarize what is happening:

   a. (i) $8^{\frac{1}{3}}$ (ii) $8^{\frac{2}{3}}$ (iii) $8^{\frac{4}{3}}$ (iv) $8^{\frac{5}{3}}$
   b. (i) $4^{\frac{1}{2}}$ (ii) $4^{\frac{3}{2}}$ (iii) $4^{\frac{5}{2}}$ (iv) $4^{\frac{7}{2}}$
   c. (i) $16^{\frac{1}{4}}$ (ii) $16^{\frac{3}{4}}$ (iii) $16^{\frac{5}{4}}$ (iv) $16^{\frac{7}{4}}$
   d. Explain how to work through a question wherein you are asked to evaluate $A^{\frac{b}{c}}$

5. **<u>(CA - DESMOS)</u>** **Investigation #1**

   a. Use DESMOS to graph $y = 2^x$.
   b. Then graph $y = b^x$ and add a slider for b. Set the slider for b for $1 \leq b \leq 10$
   c. Play the slider and record observations and describe the effect of "b" on the exponential function.
   d. What is changing about the exponential function ⇒ its shape or its location?

6. **<u>(CA - DESMOS)</u>** **Investigation #2**

   a. Use DESMOS to graph $y = 2^x$.
   b. Then graph $y = b^x$ and add slider. Set the slider for b for $0 \leq b \leq 1$
   c. Play the slider and record observations and describe the effect of "b" on the exponential function. How is this different that Investigation #1?
   d. What is changing about the exponential function ⇒ its shape or its location?

7. **<u>(CA)</u>** Mr S has been given a new job contract. He will earn $50,000 in the first year of this contract and get a raise of 6% of his previous years' salary (i.e his salary grows by 6% per year).

   a. Write an equation for Mr. S's salary.
   b. Graph the function on your TI-84.
   c. What does the y-intercept represent?
   d. What would my salary be in 8 years?
   e. After how many years would my salary be $80,000?
   f. What assumption are you making as you answer questions d and e?

8. **<u>(CA)</u>** Ratio Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of $f(x) = ab^x$ for the data set.

x	2	3	4	5	6	7	8
$f(x)$	6.6	7.6	8.7	10.1	11.6	13.3	15.3

   a. Determine the "common ratio" between each pair of terms (you do this by dividing the successive y terms ==> ratio = $\frac{y_2}{y_1}$; ratio = $\frac{y_3}{y_2}$; r = $\frac{y_4}{y_3}$; etc …..
   b. This value for the common ratio is the ***base*** or b in the equation. How can you use the data set to find the value for a?
   c. Finally, what is the equation for this data set?

9. **(CA)** On January 1st, 2000, Mr S made a deposit of \$15,000 in an account pays annual interest of 7.25% on the balance annually and the account balance is modeled by $B(t) = 15{,}000(1.0725)^t$, where t is time in years since January 1st, 2000.

   a. From the equation, how do you know that the model represents a growth curve?
   b. At what rate is the deposit growing?
   c. What was the value of the deposit on Jan 1st, 2019?
   d. What is the value of the deposit now?
   e. In what year will the value of the investment exceed \$80,000?

10. **(CA)** Mr R has purchased a new car. It cost \$50,000 but its value is depreciating at an annual rate of 12% of the previous year's value.

   a. Write an equation for the value of Mr. R's car.
   b. Graph the function on your TI-84.
   c. What does the y-intercept represent?
   d. What is the value of his car in 8 years? What assumption are you making?
   e. After how many years would the value of his car be \$10,000? What assumption are you making?

11. **(CI)** Determine the intersection point of the following functions using algebraic methods. Now, verify your solutions by graphing on your TI-84.

   a. $2x - 5y = 12$ and $-4x + y = 12$
   b. $y = 2^x$ and $y = 8 - 3(2^x)$

12. **(CI)** Exponential functions can be written in the form of $f(x) = ab^x$. Write the equation of the exponential function that go through the following pairs of points:

   a. W(1, 12) and X(4, 1.5) b. $f(-1) = -144$ and $f(2) = -\frac{9}{4}$