

## IM2 Problem Set 6.3 - Exponential Functions

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> <li>How can we analyze growth or decay patterns in data sets &amp; contextual problems?</li> <li>How can we algebraically &amp; graphically summarize growth or decay patterns?</li> <li>How can we compare &amp; contrast linear and exponential models?</li> <li>How can we extend basic function concepts using exponential functions?</li> </ul>
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### Part 1 - Skills/Concepts Review

- (CI)** Exponential functions can be written in the form of  $f(x) = ab^x$ . Write the equation of the exponential function that go through the following pairs of points:
  - W(1, 12) and X(3, 108)
  - $f(2) = 2$  and  $f(5) = 128$
- (CI)** Use the exponent laws to write each expression with a single, simplified base. All exponents must be positive in your final answers.
  - $a^4 \times a^5 \times a^6$
    - $\frac{a^4}{a^{10}}$
    - $\frac{d}{d^{-5}}$
    - $(2a^2b^{-3})^2$
  - $a^4 \times a^{-5} \times a^{-3}$
    - $(3a^{-2}b^3)^{-2}$
    - $\frac{a^4 \times b^3}{b^5 \times a^{-2}}$
    - $\frac{(K^a)^b \cdot K^{ab}}{K^{4ab}}$
- (CI)** Evaluate (simplify as a number) the following:
  - $-3^2$
    - $(-3)^2$
    - $-3^{-2}$
    - $(-3)^{-2}$
    - $(3^{-2} + 3^{-1})^{-1}$
  - $(\frac{-2}{5})^2$
    - $(\frac{-2}{5})^{-2}$
    - $[(\frac{-2}{5})^{-2}]^{-1}$
    - $-(\frac{-2}{5})^2$
    - $(\frac{-2}{5})^3$
- (CA)** Youssef's mark in SEM 2 started at 40% but has been increasing by 8% every week.
  - Complete this table of values for this relationship between Youssef's mark and the number of weeks since the start of the semester.
 

Week number	0	1	2	3	4	5
Mark	40					
  - Determine his mark: (i) in week 7    (ii) in week 10    (iii) in week 15

### Part 2 - Skills/Concepts Application Problems

5. **(CA)** Mr Santowski's investments have been decreasing by 2% every month since January 1<sup>st</sup>, 2019. The total value of his investments were 250,000 USD on March 1<sup>st</sup>, 2019.

- a. Complete this table of values for this relationship between the value of Mr. S investments and the number of months since January 1<sup>st</sup>, 2019.

Month	Jan	Feb	Mar	Apr	May	June
Value			250000			

- b. Determine the value of his investments on (i) July 1<sup>st</sup>, 2019 and (ii) on Jan 1<sup>st</sup>, 2020

6. **(CI)** Solve the following equations for  $x$ .

- |                               |                            |
|-------------------------------|----------------------------|
| a. (i) $2x + 4 = 5$           | (ii) $2^x + 4 = 5$         |
| b. (i) $2x + 15 = 15.5$       | (ii) $2^x + 15 = 15.5$     |
| c. (i) $2(x + 1) + 15 = 15.5$ | (ii) $2^{x+1} + 15 = 15.5$ |
| d. (i) $2(x + 1) + 5 = -11$   | (ii) $2^{x+1} + 5 = -11$   |

7. **(CI)** Difference Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of  $f(x) = mx + b$  for the data set.

$x$	-2	-1	0	1	2	3	4
$f(x)$	-1	2	5	8	11	14	17

- a. Determine the "common difference" between each pair of terms (you do this by subtracting the successive  $y$  terms  $\Rightarrow$  difference  $= y_2 - y_1$ ;  $d = y_3 - y_2$ ;  $d = y_4 - y_3$ ; etc .....)
- b. This value for the common difference is the **slope** or  $m$  in the equation. How can you use the data set to find the value for  $b$ ?
- c. Finally, what is the equation for this data set?

8. **(CA)** A colony of 1000 ants is growing at a rate of 15% every month.

- a. How many ants will be in the colony after 10 months? What assumptions are you making?
- b. How long will it take to get a population of 7500 ants?
- c. Mr. S would like to predict the number of ants present in one week. Explain how he could do this.

9. **(CA)** Ratio Analysis of a Data Set. Mr S. gives you this data set and is asking you to analyze patterns in the data set in order to determine an equation in the form of  $f(x) = ab^x$  for the data set.

$x$	-2	-1	0	1	2	3	4
$f(x)$	$\frac{8}{9}$	$\frac{4}{3}$	2	3	4.5	6.75	10.125

- Determine the “common ratio” between each pair of terms (you do this by dividing the successive  $y$  terms  $\Rightarrow$  ratio =  $\frac{y_2}{y_1}$ ; ratio =  $\frac{y_3}{y_2}$ ;  $r = \frac{y_4}{y_3}$ ; etc .....
- This value for the common ratio is the **base** or  $b$  in the equation. How can you use the data set to find the value for  $a$ ?
- Finally, what is the equation for this data set?

10. **(CA)** Ten grams of a chemical - Mathonium - is stored in a container. The amount of Mathonium present in the container can be modeled by  $C(t) = 12.5(0.975)^t$ , where  $C$  is the amount of Mathonium, in grams, and  $t$  is time in years from 2019.

- By considering only the equation, is the amount of Mathonium increasing or decreasing over time?
- Evaluate and interpret  $C(0)$  and  $C(100)$ .
- Find the value of  $t$  where  $C(t) = 10$ . Round your final answer to the nearest year.

11. **(CA)** At the beginning of an experiment, there are 212 bacteria. The population of bacteria will double every 2 days. How many bacteria will be present in:

- 8 days
- 11 days
- 2 months
- 1 day
- 12 hours

12. **(CA)** A population of hamsters will triple every year. Initially, the population started with 10 hamsters.

- What will be the population of hamsters after 4 years? What assumptions are you making? Are these assumptions reasonable?
- How long will it take to get a population of 1500 hamsters?
- Mr. S would like to predict the number of hamsters present in 6 months. Explain how he could do this.

### Part 3 - Extension Problems

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