IM2 Problem Set 6.2 - Exponential Functions

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models? • How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CI)** Linear functions can written in the form of $f(x) = ax + b$ whereas exponential functions can be written in the form of $f(x) = ab^x$. Write the equation of the linear function as well as the equation of the exponential function that go through the following pairs of points:

   a. W(2, 4) and X(3, 8) b. W(2, 4) and X(4, 16) c. W(2, 4) and X(4, 36)

2. **(CI)** Evaluate the following numerical expressions:

   a. (i) 4^{-3} (ii) 5^{-2} (iii) 2×5^0 (iv) 10^{-2}
   b. (i) $2^{-1} + 2^2 + 2^3 - 2^{-2}$ (ii) $3^{-2} + 3^0 + 3(3^{-1} + 3^1)$

3. **(CA)** An exponential function has the form $f(x) = ab^x$ where a is the **initial value**, b is the **growth factor.** Decide if the equation can be used to model growth or decay and then write down the value of the growth factor. Finally, determine the **growth rate** implied by each of the equations.

$y = 200(1.15)^x$		
$y = 400(0.85)^x$		
$y = 100(2)^x$		
$y = 100(½)^x$		
$y = 200(1.05)^x$		
$y = 400(1.75)^x$		
$y = 100(0.75)^x$		
$y = 100(0.995)^x$		
$y = 1,000(0.30)^x$		
$y = 2500(1)^x$		

4. **(CI)** Using your knowledge of the "Exponent Laws", simplify each expression. All final answers should only have positive exponents.

   a. (i) $(2^{-2}x^{-4}y^{2})^{3}$ (ii) $(-3a^{3})^{-4} \times 2a^{4}$ (iii) $(2p^{5})^{-2} \times -3p^{-2}$

   b. (i) $\dfrac{2x^{3}y^{2} \times 4x^{2}y^{-2}}{3x^{-2}y^{3}}$ (ii) $\dfrac{a^{3}b^{3} \times a^{-1}b^{2}}{2a^{-2}b^{4}}$ (iii) $\dfrac{(2x^{-1})^{-2}y^{4}}{8x^{4}y^{5} \times 2x^{2}y^{-2}}$

5. **(CI)** A relation is defined by the following description: To generate the numbers in this relation, the **starting number** will be 200. Every subsequent number is made by always **increasing the previous number** by a factor of $\frac{3}{2}$. Create a table of values for this relation and then graph this relation. Predict an equation for this relation.

Part 2 - Skills/Concepts Application Problems

6. (CA) Mr S invests some money into two different accounts. On the first account, he invested \$7500 and earns **compound interest** of 4% on every year on this investment. The future value of his money can be modeled using the equation $y = 7500(1 + r)^{t}$, where t represents the number of years that he owns the investment. The second account earns **simple interest** of 8% every year and can be modeled as $y = 7500 + 7500rt$, where t represents the number of years that he owns the investment.

   a. What does the 7500 represent?
   b. Graph $y = 7500(1 + 0.04)^{t}$
   c. Graph $y = 7500 + 7500(0.08)t \Rightarrow$ which can be written as $y = 7500(1 + 0.08t)$
   d. Which function is exponential and which equation is linear?
   e. Determine the value of each investment in 10 years time.
   f. Use the table on your TI-84 to determine when the value of each investment has doubled.

7. **(CA)** Since January 1st, 2000, the occurrence of the disease called Mathitis has been changing according to the mathematical model $P(t) = 750(0.90)^{t}$, where t is the number of years since 2000.

   a. Enter the equation into your TI-84 and look at your data table on the calculator. Now, set your windows so that you can see the function.
   b. Why would this curve be considered a "decay curve"?
   c. Evaluate and interpret $P(19)$.
   d. Solve the equation $P(t) = 100$ using (i) your data table, and (ii) your graph.
   e. Using the homescreen of your calculator, evaluate the following:

      i. $\dfrac{P(4) - P(3)}{P(3)}$ ii. $\dfrac{P(5) - P(4)}{P(4)}$ iii. $\dfrac{P(10) - P(9)}{P(9)}$ iv. $\dfrac{P(a+1) - P(a)}{P(a)}$

8. **(CA)** From 1990 to 1997, the number of cell phone subscribers S (in thousands) in the US can be modeled by the equation $S(t) = 5535.33(1.413)^t$ where t is number of years since 1990.

   a. BEFORE you graph the function, explain how you know that this is a growth curve.
   b. What is the ***growth factor*** in this equation? At what ***rate*** does the number of subscribers increase?
   c. In what year was the number of cell phone subscribers about 31 million?
   d. In what year will the number of cell phone subscribers exceed 90 million?
   e. Estimate the number of subscribers in 2010.
   f. Do you think this model can be used to predict future number of subscribers? Explain

9. **(CA)** Your new computer was initially valued at \$1500 but its value, V in dollars, over time, t in years, is modelled by the equation $V(t) = 1500(0.82)^t$.

   a. BEFORE you graph the function, explain how you know that this is a decay curve.
   b. What is the ***decay factor*** in this equation? At what ***rate*** does the value of the computer decrease?
   c. Use the TABLE feature on your calculator to record the value of the computer in each of the first 4 years.
   d. How much will your computer be worth in 6 years?
   e. How long will it take before the value of your computer is half of its original value?

10. **(CI)** Given the function $h(x) = 4 - 2^{x+3}$.

   a. Without using your calculator, evaluate the following:
      i. $h(0)$ ii. $h(-1)$ iii. $h(-2)$ iv. $h(1)$ v. $h(2)$
   b. Find the value for x for which:
      i. $h(x) = 0$ ii. $h(x) = -4$ iii. $h(x) = -12$
   c. Will $h(x)$ ever equal 4? Why or why not?
   d. Using your answers from these three questions, sketch the function $h(x)$.
   e. Use your calculator and graph $h(x)$.

11. **(CA)** A population of 800 beetles is growing each month at a rate of 5%. Hanna wants to write an equation that equation that can be used to model the number of beetles, B, as a function of the number of months, $n \Rightarrow$ so she wants an equation for $B(n)$.

   a. Shivani says that the equation includes the 5%, so she writes $B(n) = 500(0.05)^n$. Paula sees the 5% and writes her equation as $B(n) = 500(5)^n$ whereas Vittoria also sees the 5%, so she writes her equation as $B(n) = 500(1.05)^n$. Which equation is correct and how did you determine the correct equation?
   b. How many beetles will there be in 8 months?
   c. When will there be 1600 beetles?

Part 3 - Extension Problems

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