IM2 Problem Set 6.1 - Exponential Functions

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models? • How can we extend basic function concepts using exponential functions?

Part 1 - Skills/Concepts Review

1. **(CI)** Expand these expressions and explain the meaning of the exponent in these expressions:

   (i) x^6 (ii) 4^5 (iii) $(2x)^4$ (iv) $(x + 3)^2$ (v) $(x - 4)^4$

2. **(CA)** Working with Functions

   a. Use your TI-84 to graph the function $f(x) = 2x$. Go to the table of values on the TI-84 and record the output values from $x = -4$ up to $x = 4$ and then sketch the function.

   b. Use your TI-84 to graph the function $g(x) = 2^x$. Go to the table of values on the TI-84 and record the output values from $x = -4$ up to $x = 4$ and then sketch the function on the same graph.

   c. Compare the table of values and comment on the patterns that you observe in the function output values.

   d. Explain what a negative exponent means and explain why.

3. **(CA)** Here are four number patterns. Describe the pattern in each set and then predict the (i) the **next** 3 terms and then (ii) the 3 terms that **came before** the first term listed.

   a. Set 1 ⇒ {...., 15, 17, 19, 21, 23,……. } Is 2019 a part of this sequence?
   b. Set 2 ⇒ { …, 20, 40, 80, 160, 320, …. } Is the number 512,000 part of this sequence?
   c. Set 3 ⇒ { ….., -14, -17, -20, -23, -26, …..} Is the number 154 part of this sequence?
   d. Set 4 ⇒ { …., 8100, 2700, 900, 300, …. } Is the number $\frac{1}{9}$ part of this sequence?

4. **(CI)** Simplify each expression. All final answers should only have positive exponents.

   a. (i) $(2x^2)(4x^3y^2)$ (ii) $(-3a^2b)(6ab^4)$ (iii) $(7p^5q^2)(-3p^{-2}q^{-3})$
   b. (i) $(2x^3y)^3$ (ii) $2(x^3y)^3$ (iii) $(3ab^2)(2a^2b^3)^2$
   c. (i) $\frac{5x^3y^2}{10xy}$ (ii) $\frac{27a^4b^2}{18a^2b^4}$ (iii) $\frac{32(2x^2)^3y^2}{128x^4y^5}$

Part 2 - Skills/Concepts Application Problems

5. **(CA)** Using both DESMOS and your graphing calculator, graph the function $f(x) = \left(\frac{1}{2}\right)^x$

   a. Use your data table on the TI-84 and complete the following data table:

x	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$										

   b. Graph the line $y = 0$. Does the graph of the function $f(x)$ ever go below $y = 0$?
   c. What is an asymptote?

6. **(CA)** Mr S invests some money into two different accounts. On the first account, he invested $5000 and earns **compound interest** of 8% on every year on this investment. The future value of his money can be modeled using the equation $y = 5000(1 + r)^t$, where t represents the number of years that he owns the investment. The second account earns **simple interest** of 10% every year and can be modeled as $y = 5000 + 5000rt$, where t represents the number of years that he owns the investment.

   a. What does the 5000 represent?
   b. Graph $y = 5000(1 + 0.08)^t$ on your TI-84 and set the windows appropriately.
   c. Graph $y = 5000 + 5000(0.1)t$ $\Rightarrow$ which can be written as $y = 5000(1 + 0.1t)$
   d. Which function is exponential and which equation is linear?
   e. Determine the value of each investment in 10 years time.
   f. Use the table on your TI-84 to determine when the value of each investment has doubled

7. **(CA)** Since January 1st, 1980, the population of MaadiVille has grown according to the mathematical model $P(t) = 7500\,(1.2)^t$, where t is the number of years since 1980.

   a. Enter the equation into your TI-84 and look at your data table on the calculator. Now, set your windows so that you can see the function.
   b. Why would this curve be considered a “growth curve”?
   c. Evaluate and interpret (i) $P(40)$ and (ii) $P(-10)$
   d. Solve the equation $P(t) = 10\,000$ using (i) your data table, and (ii) your graph.
   e. Using the homescreen of your calculator, evaluate the following:

      i. $\frac{P(4)}{P(3)}$ ii. $\frac{P(5)}{P(4)}$ iii. $\frac{P(10)}{P(9)}$ iv. $\frac{P(a+1)}{P(a)}$

8. **(CI)** Evaluate the following numerical expressions.

   a. i. $5^2 + 5^1 - 5^0$. ii. $(3 - 5)^3 - 2^{-1}$. iii. $(2^2 + 2^3)^2 + 3^2 - 4^0$
   b. i. $(-2)^3 + 3^{-2}$. ii. $-2^3 + (2^2)^3 - 78^0$. iii. $\left(\frac{-2}{5}\right)^2 + \left(\frac{-2}{5}\right)^{-2}$

9. **(CI)** Solve the following equations for x.

a. (i) $2x - 3 = 5$ (ii) $2^x - 3 = 5$
b. (i) $2(x + 1) - 13 = 3$ (ii) $2^{x+1} - 13 = 3$
c. (i) $2x + 5 = 3(2x) - 11$ (ii) $2^x + 5 = 3(2^x) - 11$

10. Generating Data Sets. Heads or Tails Activity ⇒ Modeling Exponential Growth H&T Activity.

The purpose of this activity is to provide a simple model to illustrate exponential growth of cancerous cells. In our experiment, a HEAD on a COIN TOSS represents a cancerous cell. If the COIN lands HEADS side up, the cell divides into the "parent" cell and "daughter" cell. The cancerous cells divide like this uncontrollably-without end. We will conduct 10 trials and record the number of "cancerous cells".

Exponential Growth Procedure:

a. Use either the website http://www.shodor.org/interactivate/activities/Coin/ OR https://www.random.org/coins/ to toss our coins. We will start with 2 coins. This is trial # 0.

b. Count the number of HEADS that appear (recall these are cancerous cells) For every coin with the HEAD side showing, add another coin and then record the new population. (Ex. If 5 coins land HEADS, then you add 5 more coins)

c. Repeat step b. until you are done with 10 trials.

d. Record your data in your notebooks

Trial #	0	1	2	3	4	5	6	7	8	9	10
# of coins	2										

e. Prediction #1 ⇒ What would you predict for the # of coins for trials 11 and 12? Make your prediction and then test it out.

f. Prediction #2 ⇒ If our "cancer" becomes detectable when there are 10,000 cells, how many trials of our experiment would this take?

g. Input your data into the STAT lists and graph the data set as a scatter plot. Use the TI-84 to determine the equation of the exponential model for your data set.

Part 3 - Extension Problems

11. **(CA)** Dr. Harris is offering Mr. Santowski & Mr. Smith new contract options for March. Here are the terms of the contracts being offered:

OPTION A ⇒ Here is Mr. Smith's payment option: Get paid $5,000 US per day for each day in the month of March.

OPTION B ⇒ Here is Mr. Santowski's payment option:

i. Get paid 1 piastre on the first day of March.
ii. But then on the 2nd of March, return the 1 piastre and get paid double yesterday's wage, so get 2 piastres for having worked 2 days.
iii. Now, on the 3rd of March, return the 2 piastres and get paid double yesterday's wage of 2 piastres, making it a total of 4 piastres pay for these three days.
iv. Alas, on the 4th of March, return the 4 piastres and get paid double yesterday's wage of these 4 piastres, making it a total of 8 piastres pay for these four days.
v. Oh, woe is me. On the 5th of March, I return the 8 piastres, but get paid double yesterday's wage of these 8 piastres, making it a total of 16 piastres pay for these five days.

a. Which option would you choose and why?
b. Are the salaries ever equal? If so when? If not why not?
c. How much does each Math teacher get paid by the end of March? Convert to a common currency & show your work.