

IM2 Problem Set 5.9 - Linear Functions

BIG PICTURE of this UNIT:	• What is meant by the term FUNCTIONS and how do we work with them? • mastery with working with basics & applications of linear functions • mastery with working with basics & applications of linear systems • understanding basics of function concepts and apply them to lines & linear systems
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Part 1 - Skills/Concepts Review

- The sequence of numbers 3, 12, 21, 30, is called an **arithmetic sequence** because consecutive terms increase by the **same difference** (+9 in this case).
 - Write the next 4 terms of this sequence.
 - Write a general formula that can be used to produce further terms of the sequence.
 - Hence, predict what the 100th term should be.
- Determine which sequences are arithmetic. For those that are, state the common difference and write a general formula that can be used to predict further terms:
 - 1,5,9,13,17,....
 - 3,7,13,17,23,27,....
 - 3,6,12,24,....
 - 59,48,37,26,15,....
- What is the 50th term of the arithmetic sequence 85, 102, 119, ?
- You know that $f(1) = 5$ and you also know that $f(n + 1) = f(n) + 12$, where n must be a positive integer starting with $n = 1$. Determine:
 - $f(2)$
 - $f(3)$
 - $f(4)$
 - What type of sequence have you produced?
- The sequence of numbers 9, 18, 36, 72, 144, is called an **geometric sequence** because consecutive terms increase by the **same multiple** ($\times 2$ in this case).
 - Write the next 4 terms of this sequence.
 - Write a general formula that can be used to produce further terms of the sequence.
 - Hence, predict what the 20th term should be.
- Determine which sequences are geometric. For those that are, state the common multiple and write a general formula that can be used to predict further terms:
 - 15, 26, 37, 48,....
 - 5, 15, 45, 135 ,....
 - 3, 9, 81, 6561,....
 - 6000, 3000, 1500, 750, 375,....
- What is the 10th term of the geometric sequence 316, 1106, 3871.5, ?

8. You know that $f(1) = 243$ and you also know that $f(n + 1) = \frac{1}{3} f(n)$, where n must be a positive integer starting with $n = 1$. Determine:
- a. $f(2)$ b. $f(3)$ c. $f(4)$ d What type of sequence have you produced?

Part 2 - Skills/Concepts Application Problems

9. Terry invests \$300 in a GIC (guaranteed investment certificate) that pays 6% **simple interest** per year.
- a. Determine the value of his investment in each of the first five years that he owns this GIC.
- b. When will his investment be worth \$732?
10. Phil invests \$5000 in a high interest savings account and earns 3.5% **simple interest** per year.
- a. Determine the value of his investment in each of the first five years..
- b. When will his investment be worth \$7800?
11. Janice gets a job that pays \$15.25/hour. Her boss has promised to raise her wages by \$0.15/h after each month of work. How long will Janice have to work before she doubles her wages?
12. Sam invested \$5000 in a GIC earning 3.5% **compounded interest** per year. (Compounding means that the interest get added to the amount invested, so that the next year Sam's interest is based upon the original amount in addition to the interest he earned the first year).
- a. Determine the value of his investment in each of the first five years..
- b. When will his investment be worth \$7800?
13. A doctor makes observations of a bacterial culture at regular time intervals, as shown on the table.
- a. Determine the amount of bacteria in each of the next 5 observations.
- b. After how many observations will the number of bacteria first exceed 1 000 000?
- c. What assumptions are you making in your answers?
- | Observation | Number of Bacteria |
|-------------|--------------------|
| 1 | 5 120 |
| 2 | 7 680 |
| 3 | 11 520 |
| 4 | 17 280 |
14. Janice gets a job that pays \$15.25/hour. Her boss has promised to raise her wages by 1.25% after each month of work. How long will Janice have to work before she doubles her wages?
15. What would you rather earn on a \$10,000 investment - an annual rate of 5% **simple interest** or an annual rate of 3.5% **compounded interest**? Show the mathematical reasoning that leads to your conclusion.

Part 3 - Extension Problems

16. The first term of an arithmetic sequence is 13. Two other terms of the sequence are 37 and 73. The common difference between consecutive terms is an integer. Determine all possible values of the 100th term.
17. Given the geometric sequence with the first term being 1 and the common multiple being $\frac{1}{2}$,
- determine the next 5 terms,
 - determine the sum of the first two terms
 - Determine the sum of the first 3 terms, the first 4 terms, the first 5 terms
 - What seems to be happening to the sum as you add more and more terms?
18. A square has a side length of 12 cm. The midpoints of the square are joined creating a smaller square and four triangles. If you continue this process, what will be the total area of the shaded region in stage 6? In stage 12? In stage 1000?

