IM2 Problem Set 5.5 - Linear Functions

BIG PICTURE of this UNIT:	• What is meant by the term FUNCTIONS and how do we work with them? • mastery with working with basics & applications of linear functions • mastery with working with basics & applications of linear systems • understanding basics of function concepts and apply them to lines & linear systems

Part 1 - Skills/Concepts Review

1. Use mini whiteboards to graph the following linear relations:

   a. $f(x) = 5 - 2x$ b. $g(x) = -\frac{1}{3}x + 4$ c. $2x - 3y = 12$ d. $y = 2$ e. $x = -3$

2. Given the function $g(x) = 4 - \frac{1}{2}x$.

   a. Graph this function.
   b. Determine the domain and range of $g(x)$.
   c. Factor the equation $g(x) = -\frac{1}{2}x + 4$
   d. Determine the x- and y-intercepts.
   e. Solve $g(x) = -8$
   f. Evaluate $g(-2)$
   g. Write this equation in standard form.

3. Given the linear functions $f(x) = 2x + 7$ and $g(x) = 5 - x$.

   a. Explain how you know that these lines MUST intersect.
   b. WITHOUT graphing, determine the intersection point.

4. If apples cost 3 LE per apple and oranges cost 4 LE per orange, how many apples and oranges can I buy for 70 LE? Given your answer(s), explain what the idea of a “unique solution” means.

Part 2 - Skills/Concepts Application Problems

5. Solve the linear system $y = 2x - 4$ and $3x + 2y = 15$ using the substitution method.

6. Solve the following linear system by elimination.

    a. L_1: $2x + 5y = 4$ and L_2: $-2x + y = 8$
    b. L_1: $x + y = 5$ and L_2: $3x + y = 11$

7. FIXIT Pool Repair Service charges $50 for a service call and $40/hour for labour. Oasis Pools charges $30 for a service call plus $45/hour for labour.

    a. The cost of repairing your pool can be modeled by linear functions. Write linear functions that model the cost of service provided by each of the pool repair companies.
    b. Hence or otherwise, find the number of hours for a repair job for which both companies would charge the same amount.

8. Max is training for the upcoming Track & Field season. He needs to design a 45 minute daily workout using a combination of a stationary bike and a treadmill. To be in top shape, he needs to burn 400 calories in his workout. On a bike, he burns 8 cal/min and on the treadmill he burns 10 cal/min.

    a. Write two linear equations that could be used to model this problem. Start by defining the variables that you would need in your equations.
    b. How many minutes should he train on each piece of equipment?
    c. Suggest a domain and range for this word problem.

9. Water usually boils at 100℃. At higher altitudes however, water boils at lower temperatures. Suppose that water boils at 96.5℃ at an altitude of 1000m and boils at 93.0℃ at an altitude of 2000m.

    a. Amir proposes a linear relationship between altitude and boiling point. Which one would be the independent variable? Which one would be the dependent variable?
    b. Write an equation modeling the relationship between altitude and boiling point.
    c. What does the slope mean? What does the y-intercept mean?
    d. What would be the boiling point of water at South Base Camp on Mount Everest (elevation 17,600m)

10. Six cups of coffee and a dozen muffins originally cost $15.35. The price of coffee increases by 10% and the price of the muffins increases by 12%. So the new cost for six coffee and a dozen muffins is $17.06. Determine the new price of one cup of coffee and the new price of one muffin.

11. A hot air balloon is presently at a height of 500 m. It develops a leak and begins to descend at a rate of 60 m/min.

    a. Create a linear model that relates the balloon's height to time.
    b. Graph this model.
    c. Use your linear model to predict the balloon's height at 5 minutes and 10 minutes.
    d. Use your linear model to determine how long it would take the balloon to reach the ground.

12. The function f is defined by $f(x) = 2x + 3$ and the function g is defined as $g(x) = 3x + 5$. Answer the following questions about these functions.

    a. Evaluate: i. $f(3) - f(2)$ ii. $f(4) - f(3)$ iii. $f(5) - f(4)$ iv. $f(a + 1) - f(a)$
    What observation do you make and why does this happen?

    b. Evaluate: i. $g(3) - g(2)$ ii. $g(4) - g(3)$ iii. $g(5) - g(4)$ iv. $g(a + 1) - g(a)$
    What observation do you make and why does this happen?

    c. Evaluate i. $g(f(-5))$ ii. $f(g(2))$ iii. $f(g(x))$ iv. $g(f(x))$.

Part 3 - Extension Problems

13. If $f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$,

    a. find the value of $f(10)$ - $f(-3)$.
    b. sketch $y = f(x)$

14. For all real numbers x and y, we will define $x♠y$ as the following $x♠y = (x + y)(x - y)$. What is the value of $3♠(4♠5)$?