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Topic 5: Calculus

Concepts

Essential understandings:

Calculus describes rates of change between two variables and the accumulation of limiting areas. Understanding these rates of change and accumulations allow us to model, interpret and analyze real-world problems and situations. Calculus helps us to understand the behaviour of functions and allows us to interpret the features of their graphs.

Suggested concepts embedded in this topic:

Change, patterns, relationships, approximation, generalization, space, modelling.

AHL: Systems, quantity.

Content-specific conceptual understandings:

- The derivative may be represented physically as a rate of change and geometrically as the gradient or slope function.
- Areas under curves can be approximated by the sum of the areas of rectangles which may be calculated even more accurately using integration.
- Examining rates of change close to turning points helps to identify intervals where the function increases/decreases, and identify the concavity of the function.
- Numerical integration can be used to approximate areas in the physical world.
- Mathematical modelling can provide effective solutions to real-life problems in optimization by maximizing or minimizing a quantity, such as cost or profit.
- Derivatives and integrals describe real-world kinematics problems in two and three-dimensional space by examining displacement, velocity and acceleration.

AHL

- Some functions may be continuous everywhere but not differentiable everywhere.

Syllabus

- A finite number of terms of an infinite series can be a general approximation of a function over a limited domain.
- Limits describe the output of a function as the input approaches a certain value and can represent convergence and divergence.
- Examining limits of functions at a point can help determine continuity and differentiability at a point.

SL content

Recommended teaching hours: 28

The aim of the SL content in the calculus topic is to introduce students to the concepts and techniques of differential and integral calculus and their applications.

Throughout this topic students should be given the opportunity to use technology such as graphing packages and graphing calculators to develop and apply their knowledge of calculus.

Sections SL5.1 to SL5.5 are content common to both Mathematics: analysis and approaches and Mathematics: applications and interpretation.

SL 5.1

Content	Guidance, clarification and syllabus links
Introduction to the concept of a limit.	<p>Estimation of the value of a limit from a table or graph.</p> <p>Not required: Formal analytic methods of calculating limits.</p>
Derivative interpreted as gradient function and as rate of change.	<p>Forms of notation: $\frac{dy}{dx}$, $f'(x)$, $\frac{dV}{dr}$ or $\frac{ds}{dt}$ for the first derivative.</p> <p>Informal understanding of the gradient of a curve as a limit.</p>

Connections

Links to other subjects: Marginal cost, marginal revenue, marginal profit, market structures (economics); kinematics, induced emf and simple harmonic motion (physics); interpreting the gradient of a curve (chemistry)

Aim 8: The debate over whether Newton or Leibnitz discovered certain calculus concepts; how the Greeks' distrust of zero meant that Archimedes' work did not lead to calculus.

International-mindedness: Attempts by Indian mathematicians (500-1000 CE) to explain division by zero.

TOK: What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? Is intuition a valid way of knowing in mathematics?

Use of technology: Spreadsheets, dynamic graphing software and GDC should be used to explore ideas of limits, numerically and graphically. Hypotheses can be formed and then tested using technology.

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Content	Guidance, clarification and syllabus links
Increasing and decreasing functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$, $f'(x) < 0$.	Identifying intervals on which functions are increasing ($f'(x) > 0$) or decreasing ($f'(x) < 0$).

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SL 5.3

Content	Guidance, clarification and syllabus links
Derivative of $f(x) = ax^n$ is $f'(x) = anx^{n-1}$, $n \in \mathbb{Z}$ The derivative of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$ where all exponents are integers.	

Connections

TOK: The seemingly abstract concept of calculus allows us to create mathematical models that permit human feats such as getting a man on the Moon. What does this tell us about the links between mathematical models and reality?

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Content	Guidance, clarification and syllabus links
Tangents and normals at a given point, and their equations.	Use of both analytic approaches and technology.

Connections

Links to other subjects: Instantaneous velocity and optics, equipotential surfaces (physics); price elasticity (economics).

TOK: In what ways has technology impacted how knowledge is produced and shared in mathematics? Does technology simply allow us to arrange existing knowledge in new and different ways, or should this arrangement itself be considered knowledge?

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SL 5.5

Content	Guidance, clarification and syllabus links
<p>Introduction to integration as anti-differentiation of functions of the form $f(x) = ax^n + bx^{n-1} + \dots$,</p> <p>where $n \in \mathbb{Z}$, $n \neq -1$</p>	<p>Students should be aware of the link between anti-derivatives, definite integrals and area.</p>
<p>Anti-differentiation with a boundary condition to determine the constant term.</p>	<p>Example: If $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$</p> <p>when $x = 1$, then $y = x^3 + \frac{1}{2}x^2 + 8.5$.</p>
<p>Definite integrals using technology.</p> <p>Area of a region enclosed by a curve $y = f(x)$</p> <p>and the x-axis, where $f(x) > 0$.</p>	<p>Students are expected to first write a correct expression before calculating the area, for example $\int_2^6 (3x^2 + 4)dx$.</p> <p>The use of dynamic geometry or graphing software is encouraged in the development of this concept.</p>

Connections

Other contexts: Velocity-time graphs

Links to other subjects: Velocity-time and acceleration-time graphs (physics and sports exercise and health science)

TOK: Is it possible for an area of knowledge to describe the world without transforming it?

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Content	Guidance, clarification and syllabus links
Derivative of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, e^x and $\ln x$.	
Differentiation of a sum and a multiple of these functions.	
The chain rule for composite functions.	Example: $f(x) = e^{(x^2+2)}$, $f(x) = \sin(3x - 1)$
The product and quotient rules.	Link to: composite functions (SL2.5).

Connections

Links to other subjects: Uniform circular motion and induced emf (physics).**TOK:** What is the role of convention in mathematics? Is this similar or different to the role of convention in other areas of knowledge?
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Content	Guidance, clarification and syllabus links
The second derivative.	Use of both forms of notation, $\frac{d^2y}{dx^2}$ and $f''(x)$.
Graphical behaviour of functions, including the relationship between the graphs of f , f' and f'' .	Technology can be used to explore graphs and calculate the derivatives of functions.
	Link to: function graphing skills (SL2.3).

Connections

Links to other subjects: Simple harmonic motion (physics).

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Content	Guidance, clarification and syllabus links
Local maximum and minimum points. Testing for maximum and minimum.	Using change of sign of the first derivative or using sign of the second derivative where $f''(x) > 0$ implies a minimum and $f''(x) < 0$ implies a maximum.
Optimization.	Examples of optimization may include profit, area and volume.
Points of inflexion with zero and non-zero gradients.	At a point of inflexion, $f''(x) = 0$ and changes sign (concavity change), for example $f''(x) = 0$ is not a sufficient condition for a point of inflexion for $y = x^4$ at $(0, 0)$. Use of the terms “concave-up” for $f''(x) > 0$, and “concave-down” for $f''(x) < 0$.

Connections**Other contexts:** Profit, area, volume.**Links to other subjects:** Velocity-time graphs, simple harmonic motion graphs and kinematics (physics); allocative efficiency (economics).**TOK:** When mathematicians and historians say that they have explained something, are they using the word “explain” in the same way?
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Content	Guidance, clarification and syllabus links
Kinematic problems involving displacement s , velocity v , acceleration a and total distance travelled.	$v = \frac{ds}{dt}; \quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ <p>Displacement from t_1 to t_2 is given by $\int_{t_1}^{t_2} v(t) dt$.</p> <p>Distance between t_1 to t_2 is given by $\int_{t_1}^{t_2} v(t) dt$.</p> <p>Speed is the magnitude of velocity.</p>

Connections

Links to other subjects: Kinematics (physics).

International-mindedness: Does the inclusion of kinematics as core mathematics reflect a particular cultural heritage? Who decides what is mathematics?

TOK: Is mathematics independent of culture? To what extent are we people aware of the impact of culture on what we they believe or know?

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Syllabus

SL 5.10

Content	Guidance, clarification and syllabus links
Indefinite integral of x^n ($n \in \mathbb{Q}$), $\sin x$, $\cos x$, $\frac{1}{x}$ and e^x .	$\int \frac{1}{x} dx = \ln x + C$
The composites of any of these with the linear function $ax + b$.	Example: $f(x) = \cos(2x + 3) \Rightarrow f(x) = \frac{1}{2} \sin(2x + 3) + C$
Integration by inspection (reverse chain rule) or by substitution for expressions of the form: $\int kg'(x)f(g(x))dx$.	Examples: $\int 2x(x^2 + 1)^4 dx$, $\int 4x \sin x^2 dx$, $\int \frac{\sin x}{\cos x} dx$.

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Content	Guidance, clarification and syllabus links
Definite integrals, including analytical approach.	$\int_a^b g(x) dx = g(b) - g(a).$ <p>The value of some definite integrals can only be found using technology.</p> <p>Link to: definite integrals using technology (SL5.5).</p>
<p>Areas of a region enclosed by a curve $y = f(x)$ and the x-axis, where $f(x)$ can be positive or negative, without the use of technology.</p> <p>Areas between curves.</p>	<p>Students are expected to first write a correct expression before calculating the area.</p> <p>Technology may be used to enhance understanding of the relationship between integrals and areas.</p>

Connections

International-mindedness: Accurate calculation of the volume of a cylinder by Chinese mathematician Liu Hui; Ibn Al Haytham: first mathematician to calculate the integral of a function, in order to find the volume of a paraboloid.

TOK: Consider $f(x) = \frac{1}{x}$, $1 \leq x \leq \infty$. An infinite area sweeps out a finite volume. Can this be reconciled with our intuition? Do emotion and intuition have a role in mathematics?

Enrichment: Exploring numerical integration techniques such as Simpson's rule or the trapezoidal rule.

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