Connections

TOK: Are there differences in terms of value that different cultures ascribe to mathematics, or to the relative value that they ascribe to different areas of knowledge?

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AHL 2.16

Content	Guidance, clarification and syllabus links
The graphs of the functions, $y = \|f(x)\|$ and $y = f(\|x\|)$, $y = \frac{1}{f(x)}$, $y = f(ax+b)$, $y = [f(x)]^2$.	Dynamic graphing packages could be used to investigate these transformations.
Solution of modulus equations and inequalities.	**Example:** $\|3x\arccos(x)\| > 1$

Connections

International-mindedness: The Bourbaki group analytic approach versus Mandlebrot visual approach.

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Topic 3: Geometry and trigonometry

Concepts

Essential understandings:

Geometry and trigonometry allows us to quantify the physical world, enhancing our spatial awareness in two and three dimensions. This topic provides us with the tools for analysis, measurement and transformation of quantities, movements and relationships.

Suggested concepts embedded in this topic:

Generalization, space, relationships, equivalence, representation,

AHL: Quantity, Modelling.

Content-specific conceptual understandings:

- The properties of shapes depend on the dimension they occupy in space.
- Volume and surface area of shapes are determined by formulae, or general mathematical relationships or rules expressed using symbols or variables.
- The relationships between the length of the sides and the size of the angles in a triangle can be used to solve many problems involving position, distance, angles and area.
- Equivalent measurement systems, such as degrees and radians, can be used for angles to facilitate ease of calculation.
- Different representations of the values of trigonometric relationships, such as exact or approximate, may not be equivalent to one another.
- The trigonometric functions of angles may be defined on the unit circle, which can visually and algebraically represent the periodic or symmetric nature of their values.

AHL

- Position and movement can be modelled in three-dimensional space using vectors.
- The relationships between algebraic, geometric and vector methods can help us to solve problems and quantify those positions and movements.

SL content

Recommended teaching hours: 25

The aim of the SL content of the geometry and trigonometry topic is to introduce students to geometry in three dimensions and to non right-angled trigonometry. Students will explore the circular functions and use properties and identities to solve problems in abstract and real-life contexts.

Throughout this topic students should be given the opportunity to use technology such as graphing packages, graphing calculators and dynamic geometry software to develop and apply their knowledge of geometry and trigonometry.

On examination papers, radian measure should be assumed unless otherwise indicated.

Sections SL3.1 to SL3.3 are content common to both Mathematics: analysis and approaches and Mathematics: applications and interpretation.

SL 3.1

Content	Guidance, clarification and syllabus links
The distance between two points in three-dimensional space, and their midpoint. Volume and surface area of three-dimensional solids including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an angle between two intersecting lines or between a line and a plane.	In SL examinations, only right-angled trigonometry questions will be set in reference to three-dimensional shapes. In problems related to these topics, students should be able to identify relevant right-angled triangles in three-dimensional objects and use them to find unknown lengths and angles.

Connections

Other contexts: Architecture and design.

Links to other subjects: Design technology; volumes of stars and inverse square law (physics).

TOK: What is an axiomatic system? Are axioms self evident to everybody?

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SL 3.2

Content	Guidance, clarification and syllabus links
Use of sine, cosine and tangent ratios to find the sides and angles of right-angled triangles.	In all areas of this topic, students should be encouraged to sketch well-labelled diagrams to support their solutions. **Link to:** inverse functions (SL2.2) when finding angles.
The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. The cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$. Area of a triangle as $\frac{1}{2}ab\sin C$.	This section does not include the ambiguous case of the sine rule.

Connections

Other contexts: Triangulation, map-making.

Links to other subjects: Vectors (physics).

International-mindedness: Diagrams of Pythagoras' theorem occur in early Chinese and Indian manuscripts. The earliest references to trigonometry are in Indian mathematics; the use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.

TOK: Is it ethical that Pythagoras gave his name to a theorem that may not have been his own creation? What criteria might we use to make such a judgment?

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SL 3.3

Content	Guidance, clarification and syllabus links
Applications of right and non-right angled trigonometry, including Pythagoras's theorem. Angles of elevation and depression. Construction of labelled diagrams from written statements.	Contexts may include use of bearings.

Connections

Other contexts: Triangulation, map-making, navigation and radio transmissions. Use of parallax for navigation.

Links to other subjects: Vectors, scalars, forces and dynamics (physics); field studies (sciences group subjects)

Aim 8: Who really invented Pythagoras's theorem?

Aim 9: In how many ways can you prove Pythagoras's theorem?

International-mindedness: The use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's gravity.

TOK: If the angles of a triangle can add up to less than 180°, 180° or more than 180°, what does this tell us about the nature of mathematical knowledge?

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SL 3.4

Content	Guidance, clarification and syllabus links
The circle: radian measure of angles; length of an arc; area of a sector.	Radian measure may be expressed as exact multiples of π, or decimals.

Connections

Links to other subjects: Diffraction patterns and circular motion (physics).

International-mindedness: Seki Takakazu calculating π to ten decimal places; Hipparchus, Menelaus and Ptolemy; Why are there 360 degrees in a complete turn? Links to Babylonian mathematics.

TOK: Which is a better measure of angle: radian or degree? What criteria can/do/should mathematicians use to make such decisions?

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SL 3.5

Content	Guidance, clarification and syllabus links
Definition of $\cos\theta$, $\sin\theta$ in terms of the unit circle.	Includes the relationship between angles in different quadrants. **Examples:** $\cos x = \cos(-x)$ $\tan(3\pi - x) = -\tan x$ $\sin(\pi + x) = -\sin x$
Definition of $\tan\theta$ as $\frac{\sin\theta}{\cos\theta}$.	The equation of a straight line through the origin is $y = x\tan\theta$, where θ is the angle formed between the line and positive x-axis.
Exact values of trigonometric ratios of 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$ and their multiples.	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, $\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$, $\tan\ 210° = \frac{\sqrt{3}}{3}$
Extension of the sine rule to the ambiguous case.	

Connections

International-mindedness: The first work to refer explicitly to the sine as a function of an angle is the Aryabhatiya of Aryabhata (ca 510).

TOK: Trigonometry was developed by successive civilizations and cultures. To what extent is mathematical knowledge embedded in particular traditions or bound to particular cultures? How have key events in the history of mathematics shaped its current form and methods?

Enrichment: The proof of Pythagoras' theorem in three dimensions**.**

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SL 3.6

Content	Guidance, clarification and syllabus links
The Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$. Double angle identities for sine and cosine.	Simple geometrical diagrams and dynamic graphing packages may be used to illustrate the double angle identities (and other trigonometric identities).
The relationship between trigonometric ratios.	**Examples:** Given $\sin\theta$, find possible values of $\tan\theta$, (without finding θ). Given $\cos x = \frac{3}{4}$ and x is acute, find $\sin 2x$, (without finding x).

Connections

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SL 3.7

Content	Guidance, clarification and syllabus links
The circular functions $\sin x$, $\cos x$, and $\tan x$; amplitude, their periodic nature, and their graphs Composite functions of the form $f(x) = a\sin(b(x + c)) + d$.	Trigonometric functions may have domains given in degrees or radians. **Examples:** $f(x) = \tan\left(x - \frac{\pi}{4}\right)$, $f(x) = 2\cos(3(x - 4)) + 1$.
Transformations.	**Example:** $y = \sin x$ used to obtain $y = 3\sin 2x$ by a stretch of scale factor 3 in the y direction and a stretch of scale factor $\frac{1}{2}$ in the x direction. **Link to:** transformations of graphs (SL2.11).
Real-life contexts.	**Examples:** height of tide, motion of a Ferris wheel. Students should be aware that not all regression technology produces trigonometric functions in the form $f(x) = a\sin(b(x + c)) + d$.

Connections

Links to other subjects: Simple harmonic motion (physics).

TOK: Music can be expressed using mathematics. What does this tell us about the relationship between music and mathematics?

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SL 3.8

Content	Guidance, clarification and syllabus links
Solving trigonometric equations in a finite interval, both graphically and analytically.	**Examples:** $2\sin x = 1, \quad 0 \le x \le 2\pi$ $2\sin 2x = 3\cos x, \quad 0° \le x \le 180°$ $2\tan(3(x-4)) = 1, \quad -\pi \le x \le 3\pi$
Equations leading to quadratic equations in $\sin x, \ \cos x$ or $\tan x$.	**Examples:** $2\sin^2 x + 5\cos x + 1 = 0$ for $0 \le x \le 4\pi$, $2\sin x = \cos 2x, \quad -\pi \le x \le \pi$ **Not required:** The general solution of trigonometric equations.

Connections

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AHL content

Recommended teaching hours: 26

The aim of the AHL content in the geometry and trigonometry topic is to extend and build upon the aims, concepts and skills from the SL content. It further explores the circular functions, introduces some important trigonometric identities, and introduces vectors in two and three dimensions. This will facilitate problem-solving involving points, lines and planes.

On examination papers radian measure should be assumed unless otherwise indicated.