Syllabus content

Topic 1: Number and algebra

Concepts

Essential understandings:

Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Suggested concepts embedded in this topic:

Generalization, representation, modelling, equivalence, patterns, quantity

AHL: Validity, systems.

Content-specific conceptual understandings:

- Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis and interpretation.
- Different representations of numbers enable equivalent quantities to be compared and used in calculations with ease to an appropriate degree of accuracy.
- Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities.
- Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.
- Logarithm laws provide the means to find inverses of exponential functions which model real-life situations.
- Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.
- The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.

AHL

- Proof serves to validate mathematical formulae and the equivalence of identities.
- Representing partial fractions and complex numbers in different forms allows us to easily carry out seemingly difficult calculations.
- The solution for systems of equations can be carried out by a variety of equivalent algebraic and graphical methods.

SL content

Recommended teaching hours: 19

The aim of the SL content of the number and algebra topic is to introduce students to numerical concepts and techniques which, combined with an introduction to arithmetic and geometric sequences and series, can be used for financial and other applications. Students will also be introduced to the formal concept of proof.

Sections SL1.1 to SL1.5 are content common to Mathematics: analysis and approaches and Mathematics: applications and interpretation.

SL 1.1

Content	Guidance, clarification and syllabus links
Operations with numbers in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.	Calculator or computer notation is **not** acceptable. For example, 5.2E30 is **not** acceptable and should be written as 5.2×10^{30}.

Connections

Other contexts: Very large and very small numbers, for example astronomical distances, sub-atomic particles in physics, global financial figures

Links to other subjects: Chemistry (Avogadro's number); physics (order of magnitude); biology (microscopic measurements); sciences group subjects (uncertainty and precision of measurement)

Syllabus

International-mindedness: The history of number from Sumerians and its development to the present Arabic system

TOK: Do the names that we give things impact how we understand them? For instance, what is the impact of the fact that some large numbers are named, such as the googol and the googolplex, while others are represented in this form?

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SL 1.2

Content	Guidance, clarification and syllabus links
Arithmetic sequences and series. Use of the formulae for the n^{th} term and the sum of the first n terms of the sequence. Use of sigma notation for sums of arithmetic sequences.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways. If technology is used in examinations, students will be expected to identify the first term and the common difference.
Applications.	Examples include simple interest over a number of years.
Analysis, interpretation and prediction where a model is not perfectly arithmetic in real life.	Students will need to approximate common differences.

Connections

International-mindedness: The chess legend (Sissa ibn Dahir); Aryabhatta is sometimes considered the “father of algebra”–compare with alKhawarizmi; the use of several alphabets in mathematical notation (for example the use of capital sigma for the sum).

TOK: Is all knowledge concerned with identification and use of patterns? Consider Fibonacci numbers and connections with the golden ratio.

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SL 1.3

Content	Guidance, clarification and syllabus links
Geometric sequences and series. Use of the formulae for the n th term and the sum of the first n terms of the sequence. Use of sigma notation for the sums of geometric sequences.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways. If technology is used in examinations, students will be expected to identify the first term and the ratio. **Link to:** models/functions in topic 2 and regression in topic 4.
Applications.	Examples include the spread of disease, salary increase and decrease and population growth.

Connections

Links to other subjects: Radioactive decay, nuclear physics, charging and discharging capacitors (physics).

TOK: How do mathematicians reconcile the fact that some conclusions seem to conflict with our intuitions? Consider for instance that a finite area can be bounded by an infinite perimeter.

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SL 1.4

Content	Guidance, clarification and syllabus links
Financial applications of geometric sequences and series: • compound interest • annual depreciation.	Examination questions may require the use of technology, including built-in financial packages. The concept of simple interest may be used as an introduction to compound interest. Calculate the real value of an investment with an interest rate and an inflation rate. In examinations, questions that ask students to derive the formula will not be set. Compound interest can be calculated yearly, half-yearly, quarterly or monthly. **Link to**: exponential models/functions in topic 2.

Connections

Other contexts: Loans.

Links to other subjects: Loans and repayments (economics and business management).

Aim 8: Ethical perceptions of borrowing and lending money.

International-mindedness: Do all societies view investment and interest in the same way?

TOK: How have technological advances affected the nature and practice of mathematics? Consider the use of financial packages for instance.

Syllabus

Enrichment: The concept of e can be introduced through continuous compounding, $\left(1+\frac{1}{n}\right)^n \to \mathrm{e}$, as $n \to \infty$, however this will not be examined.

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SL 1.5

Content	Guidance, clarification and syllabus links
Laws of exponents with integer exponents.	**Examples:** $5^3 \times 5^{-6} = 5^{-3}$, $6^4 \div 6^3 = 6$, $\left(2^3\right)^{-4} = 2^{-12}$, $\left(2x\right)^4 = 16x^4$, $2x^{-3} = \frac{2}{x^3}$.
Introduction to logarithms with base 10 and e. Numerical evaluation of logarithms using technology.	Awareness that $a^x = b$ is equivalent to $\log_a b = x$, that $b > 0$, and $\log_\mathrm{e} x = \ln x$.

Connections

Other contexts: Richter scale and decibel scale.

Links to other subjects: Calculation of pH and buffer solutions (chemistry)

TOK: Is mathematics invented or discovered? For instance, consider the number e or logarithms–did they already exist before man defined them? (This topic is an opportunity for teachers to generate reflection on “the nature of mathematics”).

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SL 1.6

Content	Guidance, clarification and syllabus links
Simple deductive proof, numerical and algebraic; how to lay out a left-hand side to right-hand side (LHS to RHS) proof. The symbols and notation for equality and identity.	**Example:** Show that $\frac{1}{4}+\frac{1}{12}=\frac{1}{3}$. Show that the algebraic generalisation of this is $\frac{1}{m+1}+\frac{1}{m^2+m}\equiv\frac{1}{m}$ LHS to RHS proofs require students to begin with the left-hand side expression and transform this using known algebraic steps into the expression on the right-hand side (or vice versa). **Example:** Show that $(x-3)^2+5\equiv x^2-6x+14$. Students will be expected to show how they can check a result including a check of their own results.

Connections

TOK: Is mathematical reasoning different from scientific reasoning, or reasoning in other Areas of Knowledge?

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SL 1.7

Content	Guidance, clarification and syllabus links
Laws of exponents with rational exponents.	$a^{\frac{1}{m}} = \sqrt[m]{a}$, if m is even this refers to the positive root. For example: $16^{\frac{3}{4}} = 8$.
Laws of logarithms. $\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ for $a, \quad x, \quad y > 0$	$y = a^x \Leftrightarrow x = \log_a y$; $\log_a a = 1, \log_a 1 = 0$, $a, \quad y \in N, \quad x \in Z$ **Link to:** introduction to logarithms (SL1.5) **Examples:** $\frac{3}{4} = \log_{16} 8$, $\log 24 = \log 8 + \log 3$ $\log 32 = 5 \log 2 \log_3 \frac{10}{4} = \log_3 10 - \log_3 4$ $\log_4 3^5 = 5 \log_4 3$ **Link to:** logarithmic and exponential graphs (SL2.9)
Change of base of a logarithm. $\log_a x = \frac{\log_b x}{\log_b a}$, for $a, \quad b, \quad x > 0$	**Examples:** $\log_4 7 = \frac{\ln 7}{\ln 4}$ $\log_{25} 125 = \frac{\log_5 125}{\log_5 25} \quad \left(= \frac{3}{2}\right)$
Solving exponential equations, including using logarithms.	**Examples:** $\left(\frac{1}{3}\right)^x = 9^{x+1}$, $2^{x-1} = 10$. **Link to:** using logarithmic and exponential graphs (SL2.9).

Connections

Links to other subjects: pH, buffer calculations and finding activation energy from experimental data (chemistry).

TOK: How have seminal advances, such as the development of logarithms, changed the way in which mathematicians understand the world and the nature of mathematics?

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SL 1.8

Content	**Guidance, clarification and syllabus links**
Sum of infinite convergent geometric sequences.	Use of $\lvert r \rvert < 1$ and modulus notation. **Link to**: geometric sequences and series (SL1.3).

Connections

TOK: Is it possible to know about things of which we can have no experience, such as infinity?

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SL 1.9

Content	**Guidance, clarification and syllabus links**
The binomial theorem: expansion of $(a+b)^n$, $n \in N$.	Counting principles may be used in the development of the theorem.
Use of Pascal's triangle and ${}^nC_r$.	${}^nC_r$ should be found using **both** the formula and technology. **Example:** Find r when ${}^6C_r = 20$, using a table of values generated with technology.

Connections

Aim 8: Ethics in mathematics–Pascal's triangle. Attributing the origin of a mathematical discovery to the wrong mathematician.

International-mindedness: The properties of "Pascal's triangle" have been known in a number of different cultures long before Pascal. (for example the Chinese mathematician Yang Hui).

TOK: How have notable individuals shaped the development of mathematics as an area of knowledge? Consider Pascal and "his" triangle.

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AHL content

Recommended teaching hours: 20

The aim of the AHL content in the number and algebra topic is to extend and build upon the aims, concepts and skills from the SL content. It introduces students to some important techniques for expansion, simplification and solution of equations. Complex numbers are introduced and students will extend their knowledge of formal proof to proof by mathematical induction, proof by contradiction and proof by counterexample.