Applying Calculus: Working with Rates of Change & Tangents & Normals

1. (CI) Given the function $f(x) = \ln(2x - 1)$, determine:

   a. The equation of the derivative.
   b. The exact value of the instantaneous rate of change at $x = 2$.
   c. At what point(s) does the function have an instantaneous rate of change of ⅓ ?

2. (CI) Given the function $f(x) = (3x + 1)\ln x$, determine:

   a. The equation of the derivative
   b. The exact value of the instantaneous rate of change at $x = e$.

3. (CI) Determine the equation of the line tangent to $y = (6x + 3)^{\frac{5}{3}}$ at the point where $x = 4$.

4. (CI) Determine the interval(s) in which $g(x) = \sin^2(x)$ is concave down on the domain of $0 \leq x \leq \pi$.

5. (CI) Find the equation of the line normal to the curve of $y = 4xe^{x^2 - 4}$ at the point (2,8).

6. (CI) For the curve defined by $f(x) = e^{-x}\cos(x)$ on the domain of $-2\pi \leq x \leq \pi$, determine:

   a. the *x*- and *y*-intercept(s);
   b. the stationary points in this domain;
   c. the "nature" of these stationary points (max/min/neither);
   d. hence, sketch the function.

7. (CI) Given the function $g(x) = \cos(x) + \frac{1}{2}\cos(2x)$, where $0 \leq x \leq 2\pi$, sketch the graph, after identifying all important features including maximum(s), minimum(s) and intervals of increase and decrease.

   a. Show that g\`(x) can be written as $g`(x) = -\sin x\,(1 + 2\cos x)$.
   b. Hence, find the stationary points of g(x).
   c. Determine the intervals of increase and decrease.
   d. Hence, sketch the function.