We will be looking at the various ways a functions can be modified by things like addition, multiplication, reflection, and other things. These transformations can apply to any function, sinusoidal, polynomial, rational etc...

PART A - Sinusoidal Transformation

1. Use DESMOS to graph the function $T(x) = \cos(x)$ in Y_1 on the domain $-360° < x < 360°$.

   a. Sketch $T(x) = \cos(x)$. Record the x-intercepts (zeroes) and **extrema** (maximum and minimum points).

   b. Then graph $f(x) = T(x) + D$ and add a slider for D. Play the slider. Describe the graph - specifically how it has been transformed relative to $T(x) = \cos(x)$.

   c. Next, graph $f(x) = AT(x)$ and add a slider for A. Play the slider. Describe the graph - specifically how it has been transformed relative to $T(x) = \cos(x)$. Do any points remain unchanged?

   d. Next, graph $f(x) = T(Bx)$ and add a slider for B, where $B > 1$. Play the slider. Describe the graph - specifically how it has been transformed relative to $T(x) = \cos(x)$.

   e. Next, graph $f(x) = T(Bx)$ and add a slider for B, but make $0 < B < 1$. Play the slider. Describe the graph - specifically how it has been transformed relative to $T(x) = \cos(x)$.

   f. Next, graph $f(x) = T(x + C)$ and add a slider for C, but make $0 < C < 360$ in steps of 10. Play the slider. Describe the graph - specifically how it has been transformed relative to $T(x) = \cos(x)$.

   g. Next, graph $f(x) = T(x - C)$ and add a slider for C, but make $0 < C < 360$ in steps of 10. Play the slider. Describe the graph - specifically how it has been transformed relative to $T(x) = \cos(x)$.

   h. KEY POINT: You have now generated the sinusoidal equation $T(x) = A \cos[B(x - C)] + D$ and you should be able to explain what transformations are implied by the parameters A, B, C and D.

PART B - Combinations of Transformations

For the next phase of this exploration we will work with a fairly unique function. It will help us visualize the impact that various transformations have on a function. This function is:

$$f(x) = \sqrt{9 - x^2}$$

If you graph this function is is the top half of a circle of radius 3, centered at the origin. Graph this function in DESMOS, we will compare each transformation back to this “parent” function.

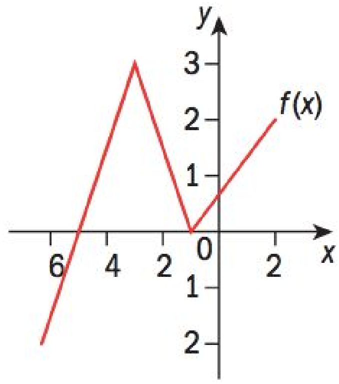
For each of the following questions, describe the transformation, and then sketch the transformations in your notes ***(DO NOT USE technology)***. Then use technology to check your work ***afterwards***.

1. $f(x) + 4$
2. $-f(x + 4)$
3. $f(2x) - 4$
4. $f(x - 2)$
5. $f(-x) + 2$
6. $\frac{1}{3}f(x)$
7. $-3f(x) - 2$
8. $f(2x)$
9. $f(-2x)$
10. $f(\frac{1}{3}x)$
11. $f(3x + 6)$
12. $2f(x) + 3$

PART C - Applications

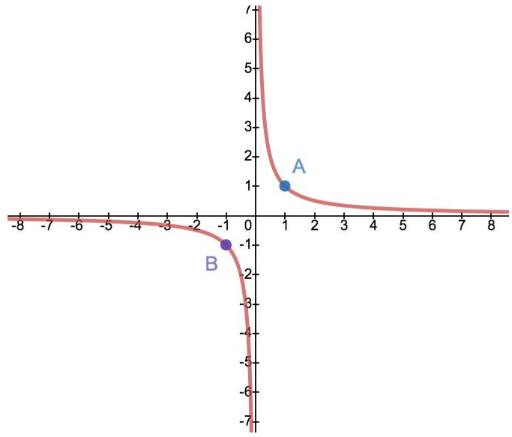
1. Copy the graph of $f(x)$. Sketch the graph of each of these functions, and state the domain and range for each.

   a. $\frac{1}{2}f(x-5)$
   b. $-f(2x)+3$
   c. $2f(x)+4$ (careful, it's a trap!)
   d. $f(2x-4)$
   e. $-f(-x)+1$



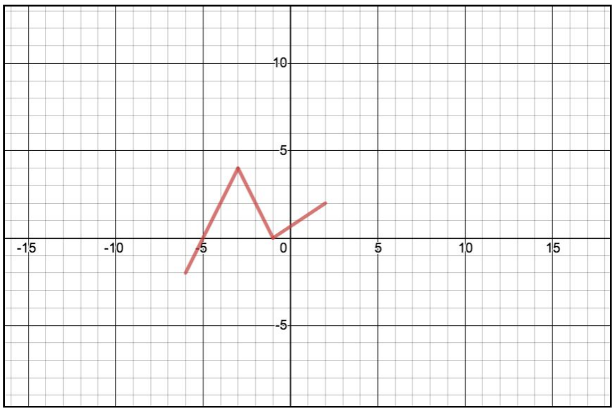
2. The graph of $f(x)$ is shown. A is the point $(1,1)$, and B is the point $(-1,-1)$. Make separate copies of the graph and draw the function after each transformation. On each graph label the new position of A as A_1, and B as B_1.

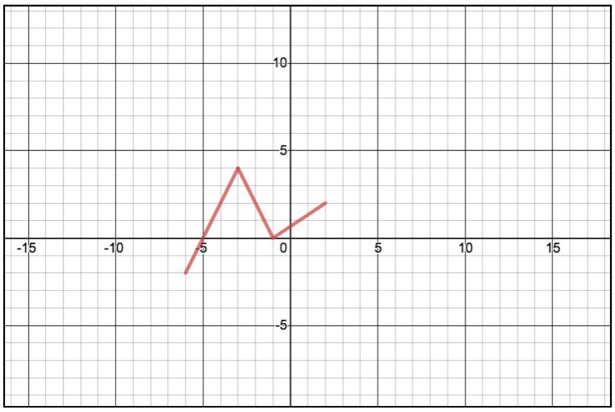
   a. $2f(x+1)$
   b. $f(x-2)+1$
   c. $3f(-x)$
   d. $2f(x)-4$
   e. $f(x-2)+3$

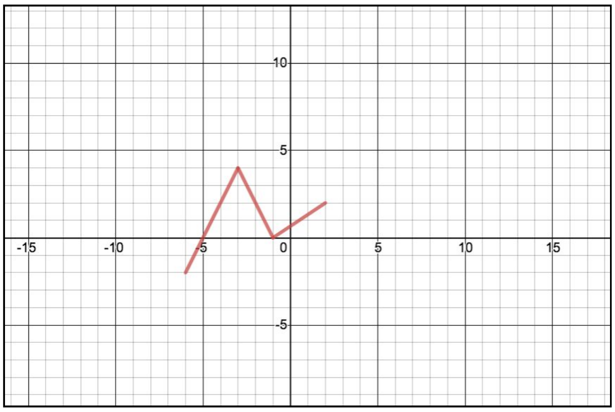


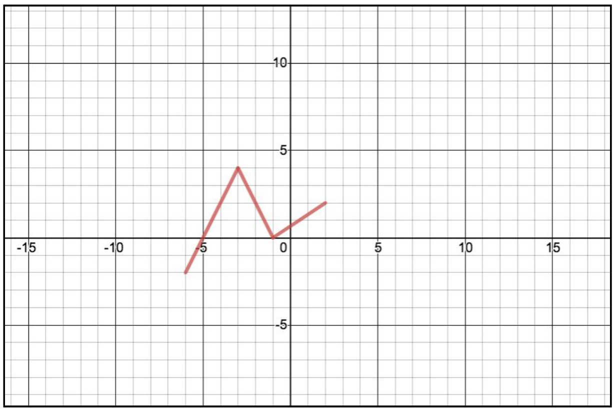
3. Consider the function $g(x) = x^2 - 6x + 2$

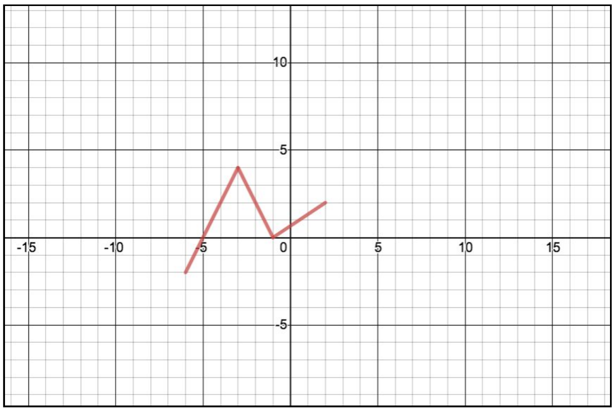
   a. Sketch the graph of f, for $-1 < x < 7$
   b. This function can also be written as $g(x) = (x-p)^2 - 7$. Write down the value of p.
   c. The graph of h_1 is obtained by reflecting the graph of f in the x-axis, followed by a vertical translation by 6 units. Show that $h_1(x) = -x^2 - 6x + 8$.
   d. The graph of h_2 is obtained by vertically stretching f by a scale factor of two followed by a horizontal shift left by 3. Show that $h_2(x) = 2x^2 - 14$.
   e. The graph of h_3 is obtained by horizontally stretching f by a scale factor of 2 followed by a Vertical shift down by 5. Show that $h_3(x) = \frac{1}{4}(x-6)^2 - 12$

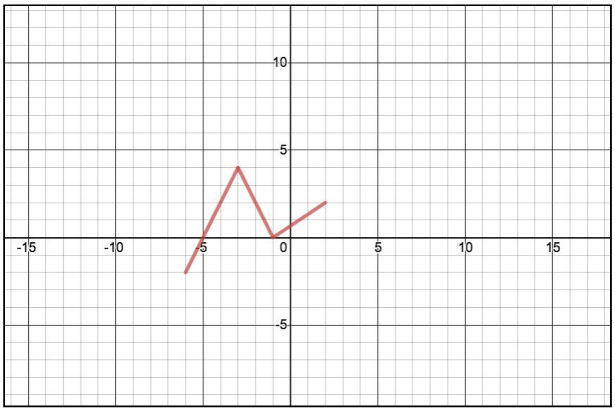

10
5
-15
-10
5
0
5
10
15
-5

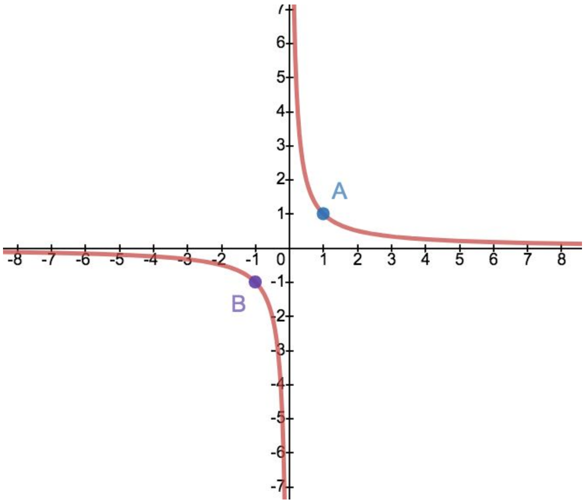

10
5
-15
-10
5
0
5
10
15
-5

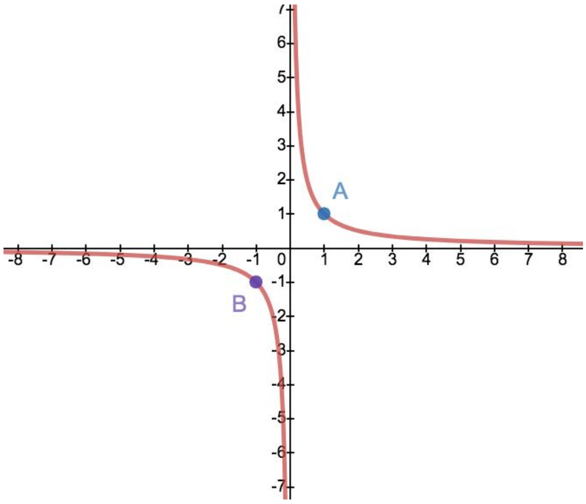

10
5
-15
-10
5
0
5
10
15
-5

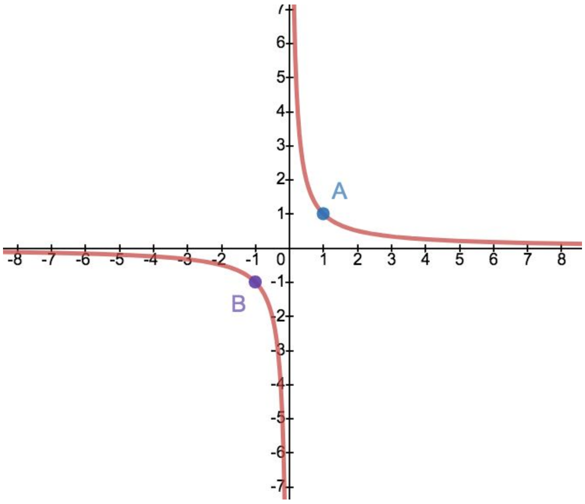

10
5
-15
-10
5
0
5
10
15
-5

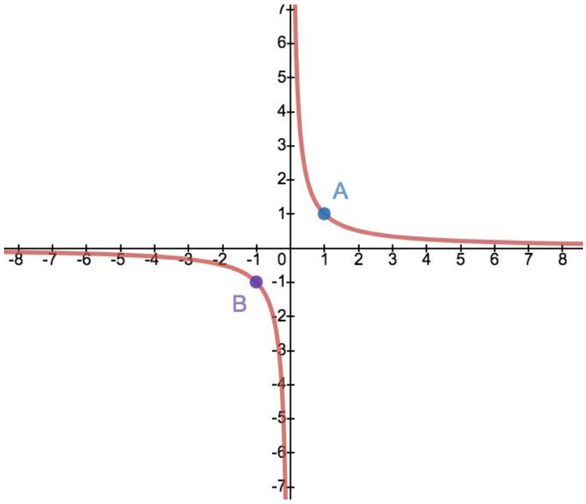

10
5
-15
-10
5
0
5
10
15
-5

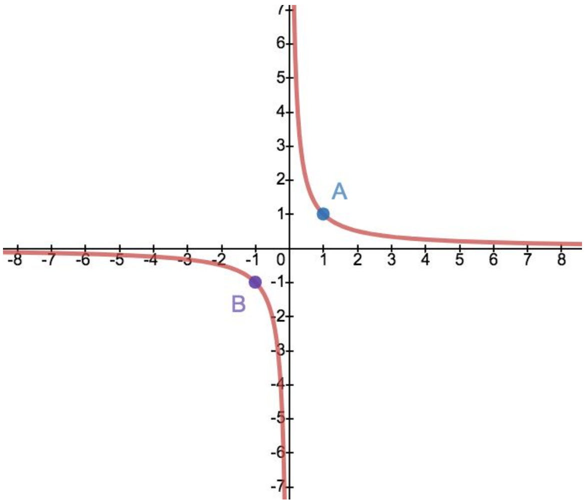

10
5
-15
-10
5
0
5
10
15
-5

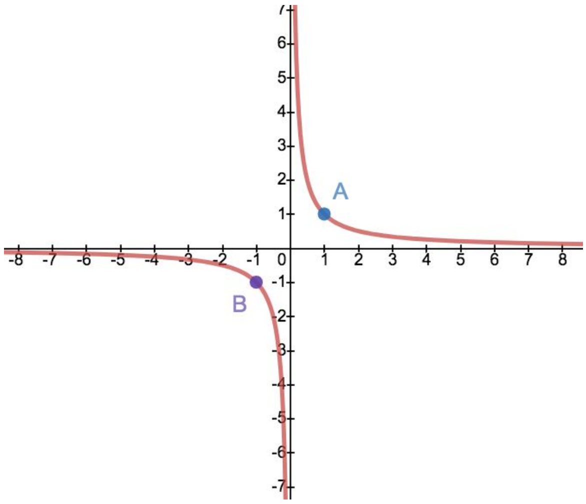

A
B


A
B


A
B


A
B


A
B


A
B