## Math SL PROBLEM SET 99

1. (CA6.3) (CI/CA) The graph of the equation $y=e^{x}\left(1-x^{2}\right)$ has two points of inflexion. Find the equations of the first and second derivatives and hence, find the coordinates of each of the inflection points. Then use a calculator to approximate the $x$ - and $y$-coordinates to three significant figures.
2. (V4.2) (CI) Find the possible value(s) of $a$ if:
a. The cosine ratio of the angle between $\boldsymbol{v}=a \boldsymbol{i}+2 \boldsymbol{j}+\boldsymbol{k}$ and $\boldsymbol{w}=3 \boldsymbol{j}-3 \boldsymbol{k}$ is $\frac{\sqrt{3}}{6}$.
b. The cosine ratio of the angle between $\boldsymbol{v}=4 \boldsymbol{i}+3 \boldsymbol{j}$ and $\boldsymbol{w}=-\boldsymbol{i}+2 \boldsymbol{j}+a \boldsymbol{k}$ is $\frac{2}{25}$.
3. (T3.6) (CA) The diagram shows $\triangle \mathrm{ABC}$. The area of $\triangle \mathrm{ABC}$ is $22 \mathrm{~cm}^{3}$.
a. Find $x$.
b. Find BC.
c. Find $\angle \mathrm{ACB}$.

4. (CA6.3) (CI) Here are the graphs of 2 functions, $f$ (in red) and $g$ (in blue). Each graph represents the DERIVATIVE of a function. For each function, prepare a graph of the ORIGINAL function.


5. (F2.4) (CI) Consider the quadratic function $g(x)=2 x^{2}-16 x+29$.
a. Express $g(x)$ in the form of $a(x-h)^{2}+k$.
b. State the coordinates of the vertex and the equation of the line of symmetry.
c. Find the exact values of the $x$-coordinates of the zeroes of $g$.
d. Is the function $g$ a one-to-one function?

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6. (V4.2-R)(CI) Points $\mathrm{A}, \mathrm{B}$, and C have position vectors $4 \boldsymbol{i}+2 \boldsymbol{j}, \boldsymbol{i}-3 \boldsymbol{j}$ and $-5 \boldsymbol{i}-5 \boldsymbol{j}$. Let D be a point on the $x$-axis such that ABCD forms a parallelogram.
a. (i) Find vector BC.
(ii) Find the position vector of D .
b. Find the angle between vectors BD and AC .

The line $L_{1}$ passes through A and is parallel to $\boldsymbol{i}+4 \boldsymbol{j}$. The line $L_{2}$ passes through B and is parallel to $2 \boldsymbol{i}+7 \boldsymbol{j}$. A vector equation of $L_{1}$ is $\boldsymbol{r}=(4 \boldsymbol{i}+2 \boldsymbol{j})+\mathrm{s}(\boldsymbol{i}+4 \boldsymbol{j})$.
c. Write down a vector equation of $L_{2}$ in the form $\boldsymbol{r}=\boldsymbol{b}+t \boldsymbol{q}$.
d. The lines $L_{1}$ and $L_{2}$ intersect at the point $P$. Find the position vector of $P$.
7. (SP5.9-R) (CA) The heights of the flowers in a flower bed are normally distributed with a mean of 43 cm and a standard deviation of 6 cm . Flowers are classified as tall flowers if they have a height that is more than 48 cm .
a. A flower is selected at random. Find the probability that this flower is a tall flower.
b. Given that this flower is tall, find the probability that it is taller than 55 cm .
c. Two flowers are selected at random. Find the probability that they are both tall.

Five hundred flowers are selected at random.
d. Find the expected value of these flowers that are tall.
e. Find the probability that at least 100 of these flowers are tall.
8. (SP5.6; SP5.8) (CA) A bag contains 10 balls. 6 of the balls are green and 4 of the balls are red. A ball is selected at random, its colour is noted, and then it is replaced.
a. What is the probability that the ball was green?

A total of 8 selections takes place. After each time, the ball is replaced.
b. What is the probability that a green ball was selected on each of the 8 occasions?
c. What is the probability that, in any order, a green ball was selected on four occasions, and a red ball was selected on four occasions?
d. What is the probability that a red ball was selected at least twice?

A game is played as follows: If a green ball is drawn then a score of -1 is awarded. If a red ball is drawn then a score of +1 is awarded. A total of 8 selections takes place.
e. (i) What is the probability of getting an odd score?
(ii) What is the probability that 0 was scored in total?
(iii) Five people play this game. What is the probability that at least one of the people scores 0 after making the 8 selections?
(iv) Given that one of the contestants scores -2 , what is the probability that the first two balls drawn were the same colour?

