Math SL PROBLEM SET 97

1. **(SP5.9) (CA)** The length of a skateboard is advertised to be 81 cm. The actual length, X metres, follows a normal distribution with a mean of 81.04 cm and a standard deviation of 1.2 cm.

   (Cirrito 17.2, p557)

   a. Find: (i) $P(X < 80)$ (ii) $P(80 < X < 82)$
   b. Given that the value of the standard deviation does not change, find the mean length necessary to guarantee that only 1% of skateboards have lengths less than 80 cm. Give your answer, accurate to four significant figures.

2. **(F2.3; CA6.1; CA6.6) (CI)** Given the function $h(x) = x^2$:

   a. Show that $g(x) = h(x - 2) - 9 = x^2 - 4x - 5$
   b. Describe fully the transformations which map $h(x)$ onto $g(x)$.
   c. For the function $y = g(x)$, use the limit definition of a derivative: $\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$ to derive the equation of the derivative of $g(x)$ from first principles.
   d. Evaluate $\int_0^6 | g(x) | \, dx$

3. **(SP5.8) (CI)** Determine the expected value for the following 4 different games:

   a. You pay \$1 to roll 2 dice. If you roll 2 odd numbers, you get \$2; If you roll 2 even numbers, you get \$2; otherwise, you get nothing.
   b. You pay \$5. You draw twice from a bag that has one \$10 bill and four \$1 bills. You get to keep the bills.

4. **(F2.6; F2.7) (CI)** Solve for x: $2^{2x} - 9(2^x) + 8 = 0$.

   (Cirrito 7.1.5, p208)

5. **(CA6.5 - N) (CA)** Find the volume of the solid generated by rotating the region bounded by the y axis and the following 2 curves $\Rightarrow$ $y^2 = x^3$ and $y^2 = 2 - x$ about the x-axis. (DESMOS may help to visualize, but you can use your TI-84) **(Oxford 9.6, p318)**

6. **(CA6.5) (CI)** The region bounded by the graphs of $y = \frac{1}{2}x + 2$, the y-axis, the x-axis and the vertical line $x = m$ has an area of exactly 45 square units. Find the value of m.

   (Cirrito 22.5, p758)

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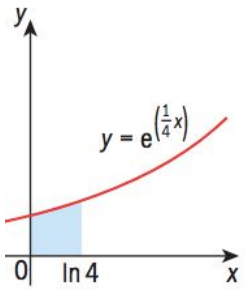
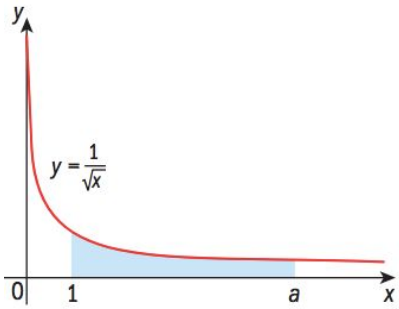
7. **(CA6.5 - N) (CI)** The following questions deal with volumes of rotation: **(Oxford 9.6, p318)**

   a. The diagram shows part of the graph of $f(x) = e^{\frac{1}{4}x}$. The shaded region between f and the x-axis from $x = 0$ to $x = \ln 4$ is rotated 360° about the x-axis.



      i. Write down a definite integral that represents the volume of the solid formed.
      ii. This volume is equal to k. Find the value of k.

   b. The shaded region in the diagram is bounded by $y = \frac{1}{\sqrt{x}}$, $x = 1$, $x = a$ and the x-axis. The shaded region is rotated 360° about the x-axis.



      i. Write down a definite integral that represents the volume of the solid formed.
      ii. The volume of the solid formed is 3π. Find the value of a.

8. **(F2.7; CA6.3) (CI)** Given the polynomial $P(w) = w^4 - 5w^2 - 36$:

(Cirrito 20.2, p649)

   a. Find all real solutions to the equation $w^4 - 5w^2 - 36 = 0$.
   b. Determine the domain intervals in which $P(w)$ is increasing and decreasing.
   c. Find the w-coordinates of all extrema and classify them.
   d. Find the w-coordinates of the inflection points.
   e. Sketch $P(w)$, labeling the information from the previous questions.