

Math SL PROBLEM SET 96

1. **(SP5.9) (CA)** The weights of a certain animal, w , are normally distributed with a mean of 36.4 kg and a standard deviation of 4.7 kg. (Cirrito 17.2, p557)
- Find the probability that when one of these animals is chosen at random it will have a weight that is: (a) 40.0 kg or less; (b) more than 45.0 kg; (c) between 32.0 kg and 41.0 kg.
 - A researcher wishes to select all animals whose weight is more than M kg. If $P(w > M)$ is to be 20%, determine the value of M .

Roll	Payout
6	\$ 4
5	\$ 2
4	\$ 1
3	\$ 0
2	\$ 0
1	\$ 0

2. **(SP5.8) (CI)** Let's say you play a game where you roll a fair die and get paid according to your roll. You have to pay \$1 to play this game. Is it worth it? What do you expect to happen in the long run?

3. **(F2.6; F2.7) (CI)** $3^{2x} + 3^x - 6 = 0$ can be written as $(3^x + m)(3^x + n)$, where $m, n \in \mathbf{Z}$. (Cirrito 7.1.5, p208)
- Find the value of m and the value of n .
 - Hence, find the exact solution for the equation $3^{2x} + 3^x - 6 = 0$ and explain why there is only one solution.

Roll	Payout
6	\$ 2
5	\$ 2
4	\$ 1
3	\$ 0
2	\$ 0
1	\$ 1.50

4. **(SP5.8) (CI)** Determine the expected value for the following games:
- You pay \$1 to roll the dice once and receive the payouts listed on the chart:
 - You pay \$1 to toss three coins. If you get all heads or all tails, you receive \$5; if not, you get nothing.

5. **(F2.6; C6.1, C6.5 - R) (CI)** Given the function $g(x) = 2e^{-x} - 1$, (Cirrito 5.3.3, p131)
- State the transformations that were applied to $y = e^x$.
 - Find the asymptote(s) and intercept(s) of g and sketch.
 - Find the equation of the line that is tangent to $f(x)$ at $x = -\ln 2$.
 - Solve for a if $\int_0^a g(x) dx = \frac{-2}{e^2}$.

6. **(F2.2; F2.2; CA6.4) (CI)** The function f is defined as $f(x) = \sqrt{4 - x}$.
- State the domain and range of f .
 - Sketch the graph of each function on a separate axis:
 - $y = f(2x)$
 - $y = f(-x)$
 - $y = -\frac{1}{2}f(x) + 2$
 - Determine the volume of the solid formed when f between $x = 0$ and $x = 4$ is rotated 360° about the x -axis.
 - Determine $\int f(x) dx$.

Math SL PROBLEM SET 96

7. **(SP5.8) (CI)** The probability of obtaining a head with a certain biased coin is $\frac{2}{3}$. The coin is tossed three times.

(Cirrito 15.1, p506)

- a. Complete the following table;

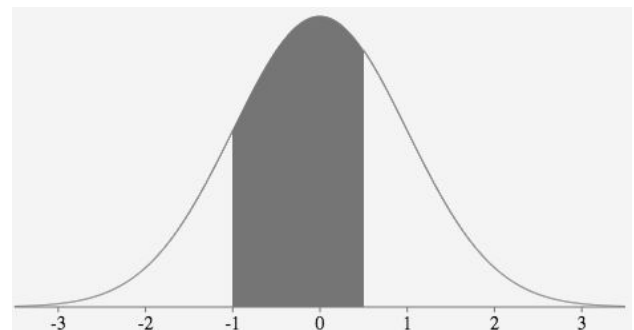
x	0	1	2	3
$P(X = x)$				

STUCO now makes a fundraiser out of this simple “game”. So you pay 4LE to play and if you get 0 heads, you will get 10LE back (representing a “payout” of 6LE). The “payouts” are summarized on the following table:

x	0	1	2	3
$P(X = x)$				
<i>payout</i>	+6	-1	-2	+2

- b. What is the expected value of the payouts? Explain why the game is not “fair”.
- c. Change the payout for getting 3 heads so that the game is now fair.

8. **(SP5.9 - R) (CA)** Reaction times of SL Math students are known to be normally distributed with a mean of 0.76 seconds and a standard deviation of 0.06 seconds. The graph included shows a standardized normal distribution, where the shaded region shows the probability of a randomly selected person having a reaction time between 0.70 seconds and 0.79 seconds.



- a. Determine $P(X \geq 0.7)$
- b. Determine $P(0.70 \leq X \leq 0.79)$
- c. Determine $P(0.70 \leq X \leq 0.79 | X \geq 0.70)$

Three percent of the SL Math students have a reaction time less than Q seconds.

- d. Find the value of Q and show this information on the graph of the distribution.