## Math SL PROBLEM SET 95

1. (F2.4) (CI) The following diagram shows part of the graph of a quadratic function $f$. The vertex is at $(1,-9)$ and the graph crosses the $y$-axis at the point $(0, c)$. The function can be written in the form $f(x)=(x-h)^{2}+k$.
a. Write down the value of $h$ and of $k$.
b. Find the value of $c$.

Let $g(x)=-(x-3)^{2}+1$. The graph of $g$ is obtained by a reflection of the graph of $f$ around the $x$-axis, followed by a translation of $p$
 units horizontally and $q$ units vertically .
c. Find the values of $p$ and $q$.
d. Find the $x$-coordinates of the points of intersection of the graphs of $f$ and $g$.
2. (T3.1) (CA) The following diagram shows a circle with centre O and radius 3 cm . Points $\mathrm{A}, \mathrm{B}$, and C lie on the circle, and $\mathrm{AOC}=1.3$ radians .
a. Find the length of arc ABC .
b. Find the area of the shaded region.

3. (SP5.7) (CA) The following table shows the probability distribution of a discrete random variable $X$. Find the value of $k$ and hence, find $E(X)$.

| $x$ | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.15 | $k$ | 0.1 | $2 k$ |

4. (CA6.3) (CI) Consider the function $g(x)=x \ln \left(x^{2}\right)$.
a. Find $g^{`}(x)$ and $g^{\prime `}(x)$.
b. Does the graph of $g$ have an inflexion point. Explain/justify your reasoning.

## Math SL PROBLEM SET 95

5. (F2.6, C6.5) (CA) Let $f(x)=2 \ln (x-3)$, for $x>3$. The following diagram shows part of the graph of $f$.
a. Find the equation of the vertical asymptote to the graph of $f$.
b. Find the $x$-intercept of the graph of $f$.
c. The region enclosed by the graph of $f$, the $x$-axis and the line $x=10$ is rotated $360^{\circ}$ about the $x$-axis. Find the volume of the solid formed.

6. (F2.5, C6.1, C6.5-R) (CI) Given the functions $f(x)=\frac{x-3}{x-1}, x \neq 1$ and $g(x)=\frac{x+4}{x+2}, x \neq-2$;
a. Rewrite $f(x)$ and $g(x)$ in the form of $y=\frac{a}{x-b}+c$ to help in identifying transformations of the parent function $y=\frac{1}{x}, x \neq 0$.
b. Sketch each of $f(x)$ and $g(x)$.
c. Hence, or otherwise, solve the inequality $f(x)>g(x)$.
d. Find the equation of the line that is tangent to $f(x)$ at $x=5$. What is the significance of the slope of the tangent line?
e. Since you have rewritten the equation for $f(x)$, evaluate $\int_{4}^{7} f(x) d x$.
7. ( $\mathbf{C 6 . 3 - \mathbf { R } ) ( \mathbf { C A } )}$ The graph shows the parabola $y=2(x-3)^{2}+2$ as well as a rectangle drawn from between the $x$-axis and the curve at Point A; so between the points where $x=0$ and $x=0.5$.
a. Determine the area of the current rectangle.
b. Point A is now free to move along the curve.

Determine a value for $x$ such that the area of the rectangle between $x=0$ and $x=a$ is a maximum area, where $0<a<3$.


## Math SL PROBLEM SET 95

8. (SP5.9) (CA) The masses of watermelons grown on a farm are normally distributed with a mean of 10 kg . The watermelons are classified as small, medium or large. A watermelon is small if its mass is less than 4 kg . Five percent of the watermelons are classified as small.
a. Find the standard deviation of the masses of the watermelons.

The following table shows the percentages of small, medium and large watermelons grown on the farm. A watermelon is large if its mass is greater than $w \mathrm{~kg}$.

| small | medium | large |
| :---: | :---: | :---: |
| $5 \%$ | $57 \%$ | $38 \%$ |

b. Find the value of $w$.

All the medium and large watermelons are delivered to a grocer.
c. The grocer selects a watermelon at random from this delivery. Find the probability that it is medium.
d. The grocer sells all the medium watermelons for $\$ 1.75$ each, and all the large watermelons for $\$ 3.00$ each. His costs on this delivery are $\$ 300$, and his total profit is $\$ 150$. Find the number of watermelons in the delivery.

