Math SL PROBLEM SET 93

1. **(CA6.2 - R) (CI)** Given the following functions, find their derivatives:

   (Cirrito 19.3, p618)

   a. $f(x) = \ln(x^2 + 4x - 2)$ b. $g(x) = 3x\cos(5x^3)$ c. $h(x) = \frac{3x^4}{e^{4x}}$

2. **(T3.5 - R) (CI)** Solve $\cos(2x) = \sin(x)$ on the domain of $-\pi \leq x \leq \pi$. **(Cirrito 10.4, p351)**

3. **(A1.3 - R) (CA)** Find the coefficient of x^4 in the expansion of $(x - 1)^2 (2x + 1)^4$.

   (Cirrito 4.1, p95)

4. **(CA6.5 - N) (CI)** Let $f(x) = (x - 2)^2$ on the domain of $x \geq 2$.

   (Oxford 9.6, p318)

   a. Determine the equation of f^{-1}.
   b. Find the volume of the solid of revolution formed by rotating the function $f^{-1}(x)$ about the x-axis between $x = 0$ and $x = 4$. (You may verify on your calculator)

5. **(T3.4; C6.1, C6.5 - R) (CI)** When a person is at rest, the blood pressure, P mm of mercury, at any time t seconds can be modeled by the equation $P(t) = -20\cos\left(\frac{5\pi}{3}t\right) + 100,\ t \geq 0$.

   (Cirrito 10.5, p361)

   a. Determine the amplitude and period of P.
   b. What is the maximum blood pressure reading that can be recorded for this person?
   c. Sketch the graph of P showing two full cycles.
   d. Find the first three times when the pressure reaches a reading of 110 mm.
   e. Find the slope of the line that is tangent to $P(t)$ at $t = 0.5$.
   f. Find $\int P(t)\,dt$.

6. **(CA6.3) (CA)** A rectangular box has height h cm, width x cm and length $2x$ cm. It is designed to have a volume equal to 1 litre (1000 cm^3).

   (Cirrito 21.4, p702)

   a. Show that $h = \frac{500}{x^2}$ cm.
   b. Find an expression for the total surface area, S cm^2, of the box in terms of x.
   c. Find the dimensions of the box that produces a minimum surface area.

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7. **(SP5.9 - R)** **(CA)** From 100 first year students writing the Biology exam, 46 of them passed while 9 were awarded "high distinction."

(**Cirrito 17.2, p571**)

   a. Assuming that the student scores were normally distributed, find the mean and standard deviation if a pass mark was 40 and "high distinction" was 75.
   b. Of those who failed, the top 50% were allowed to write a "make-up" exam. What is the lowest possible score that will allow a student to write this "make-up" exam?

8. **(V4.3)** **(CA)** In this question, distance is in kilometres and time is in hours. A small drone (remote controlled aircraft) is moving at a constant height with a speed of 15 kmh^{-1} in the direction of $\begin{pmatrix} 7 \\ 24 \\ 0 \end{pmatrix}$. At time $t = 0$, the drone is at point P with coordinates $(0, 0, 8)$.

(Oxford 12.5, p437)

   a. Show that the position vector, $\boldsymbol{r}_1$, of the drone at time t is given by

$$r_1 = \begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix} + t\begin{pmatrix} 4.2 \\ 14.4 \\ 0 \end{pmatrix}$$

At time $t = 0$, a second drone flies to intercept the first drone (to connect together for a practice recovery). The position vector of this second drone, $\boldsymbol{r}$, at time t is given by

$$r_2 = \begin{pmatrix} 36.8 \\ 85.6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -5 \\ -7 \\ 2 \end{pmatrix}.$$

   b. (i) Write down the coordinates of the starting position of the second drone.

      (ii) Find the speed of the second drone.

   c. The second drone reaches the first drone at point Q.

      (i) Find the time it takes the second drone to reach the first drone.

      (ii) Find the coordinates of Q.