Math SL PROBLEM SET 89

1. **(A1.1 - R) (CA)** Lina and Isabella begin a training program. In the first week, Isabella will run 10 km, in the second week, she will run 11 km and in the third 12 km and so on in an arithmetic progression. Lina will run 5 km in the first week and will increase her distance by 20% in each successive week. **(Cirrito 8.2.3, p262)**

   a. In what week does Lina's weekly distance first exceed Isabella's?
   b. In what week does Lina's total distance first exceed Isabella's?

2. **(A1.3 - R) (CA)** Find the constant term in the expansion of $\left(3x - \frac{2}{x}\right)^8$. **(Oxford 6.9, p184)**

3. **(F2.6 - R) (CI)** For the function $g(x) = \frac{1}{2}\ln(x + e)$, determine the (i) intercept(s) and (ii) the asymptote(s) and (iii) hence, sketch g. Then, find and sketch $g^{-1}(x)$

   (Cirrito 5.3.4, p138)

4. **(V4.4 - R) (CA)** The point (0,0) is the position of Shipple airport. The position vector, $\mathbf{r}_1$, of an aircraft, Air Farce 1, is given by $\mathbf{r}_1 = (16,12) + t(12,-5)$ where t is time in minutes since 12:00.

   a. Show that Air Farce 1
      i. is 20 km from Shipple airport at 12:00
      ii. has a speed of 13 km/hr
   b. Show that the Cartesian equation for Air Farce 1 is $5x + 12y = 144$

   The position vector of a second airplane, Spud Air 2, is given by $\mathbf{r}_2 = (23,-5) + t(2.5,6)$, where t is time since 12:00

   c. Find the angle between the 2 paths of the airplanes.
   d. Find the Cartesian equation for the flight path of Spud Air 2.
   e. Hence, or otherwise, find the coordinates of the point where the 2 flight paths cross.
   f. Given that the 2 planes are flying at the same height, show that they do NOT collide.

5. **(F2.6 - R) (CI)** A function is defined by $f(x) = 1 - e^{-x}$.

   (Cirrito 7.3, p220)

   a. Sketch the graph of f and explain how you know that an inverse function exists.
   b. Find the domain of f^{-1} and find the equation for $f^{-1}(x)$.
   c. Sketch the graph of f^{-1} on the same axes as the graph of f.
   d. Determine the equation of $(f^{-1} \circ f)(x)$. What do you notice and why does this happen?

Math SL PROBLEM SET 89

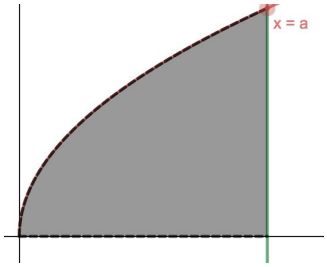
6. **(T3.5 - R) (CI)** Solve the following trigonometric equations on the domain of $0 \leq x \leq 2\pi$.

(Cirrito 10.4, p359)

a. $\sin(x) = \cos(2x)$

b. $3\tan^2(x) + \tan(x) = 2$ (CA for one ratio)

7. **(CA6.5 - R) (CI)** The area of the region enclosed by the curve $y^2 = 4ax$ and the line $x = a$ is ka^2 units. Find the value of k. **(Cirrito 22.5, p748)**



8. **(CA6.5 - N) (CA)** Determine the following volumes of rotation when: **(Oxford 9.6, p318)**

a. the region under the curve $f(x) = 2$ between $x = 0$ and $x = 5$ is rotated around the x-axis. What 3D solid is this? Use an appropriate volume formula to find volume and compare this volume (and formula) to the answer you got using the calculus.

b. the region under the curve $f(x) = 2x$ between $x = 0$ and $x = 5$ is rotated around the x-axis. What 3D solid is this? Use an appropriate volume formula to find volume and compare this volume (and formula) to the answer you got using the calculus .

c. the region under the curve $f(x) = \sqrt{4 - x^2}$ between $x = -2$ and $x = 2$ is rotated around the x-axis. What 3D solid is this? Use an appropriate volume formula to find volume and compare this volume (and formula) to the answer you got using the calculus .

9. **(SP5.9 - N) (CA)**: Over the past 50 years of the ISST track & field event, it was found that the 100m sprint times for JV girls was normally distributed with a mean of 15.6 seconds and a standard deviation of 0.24 s.

(Cirrito 17.2, p568)

a. Find the probability that a JV girl runs the 100m race in:

i. Less than 15 seconds

ii. At least 16 seconds

iii. Between 15 and 16 seconds

b. In one of the qualifying heats, 8 girls are racing. What is the probability that 6 of them will take between 15 and 16 seconds to finish the race?

c. However, the past 10 years, it has been noted that the probability of a JV girl running more than 16 seconds is 0.00621 and also that $P(X < 15) = 0.2023$. Find the mean and standard deviation of the 100m sprint times over these past 10 years.