Math SL PROBLEM SET 88

1. **(A1.2 - R) (CI)** Evaluate each of the following without a calculator: **(Cirrito 7.4, p221)**

   a. $\log_3(27^{2007})$ b. $(\log_2 5)(\log_5 12) + (\log_2 7)(\log_7 \frac{8}{3})$ c. $\frac{2^{\log_4 108}}{2^{\log_4 3}}$

2. **(CA6.6 - R) (CI)** The velocity, v, in ms^{-1} of a particle moving in a straight line is given by the function $v(t) = \sin(2t + \pi)$, where t is time in seconds. **(Cirrito 22.6, p762)**

   a. If $s(0) = 2$, find the equation for the position function, $s(t)$.
   b. Find the displacement of the particle in the first π seconds of travel. Explain the significance of your result.

3. **(SP5.9 - R) (CA)** The time taken for a student to complete an exam is normally distributed with a mean of 40 minutes and a standard deviation of 5.5 minutes. **(Cirrito 17.2, p567)**

   a. A student is selected at random. How probable is it that the student completes the exam in less than 47 minutes?
   b. Six students are selected at random. How probable is that at least 4 of them finished the exam in less than 47 minutes?
   c. The probability that a student takes between q and 47 minutes is 0.5. Find q.

4. **(SP5.9 - E) (CA)** Introducing standardized z values: Given the following means and standard deviations, determine the z-value of the given data points. **(Cirrito 17.2, p567)**

   a. If $\mu = 90$ and $\sigma = 10$, find the z-value of $x = 100$. What does this z-value MEAN?
   b. If $\mu = 45$ and $\sigma = 5$, find the z-value of $x = 40$. What does this z-value MEAN?
   c. If $\mu = 120$ and $\sigma = 18$, find the z-value of $x = 140$. What does this z-value MEAN?
   d. If $\mu = 90$ and $\sigma = 10$, find the z-value of $x = 75$. What does this z-value MEAN?

5. **(F5.9 - E) (CA)** Working with Standardized z values: Given the following means and/or standard deviations and/or z-values and/or x data values, find the unknown. **(Cirrito 17.2, p567)**

   a. If $\mu = 53$ and $\sigma = 5$ and $x = 50$, solve for z.
   b. If $\mu = 90$ and $x = 81$ and $z = -0.975$, solve for σ.
   c. If $\sigma = 55$ and $x = 200$ and $z = -1.5$, solve for μ
   d. If $\mu = 90$ and $\sigma = 20$ and $z = 1.88$, solve for x.

6. **(SP5.6 - R) (CI)** Two independent events, A and B are given where $P(A) = k$, $P(B) = k + 0.3$ and $P(A \cap B) = 0.18$. **(Oxford 3.4, p85)**

   a. Find the value of k. b. Find $P(A \cup B)$ c. Find $P(A`|B`)$

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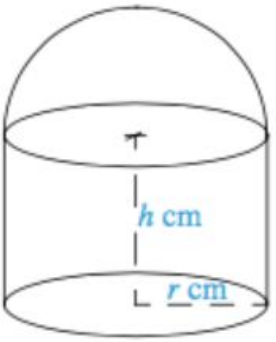
7. **(C6.3 - R) (CA)** A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can that will minimize the amount of material used in its construction. **(Cirrito 21.4, p716)**

8. **(C6.3 - R) (CA)**A closed tin is to be constructed as shown in the diagram. It is made up of a cylinder of height h cm and a radius base r cm which is surmounted by a hemispherical cap. **(Cirrito 21.4, p716)**



   a. Find an expression in terms of r and h for:
      i. its volume, V cm^3.
      ii. its surface area, A cm^2.

   b. Given that the volume is πk^3, $k > 0$, show that its surface area is given by $SA(r) = \frac{2\pi k^3}{r} + \frac{5\pi}{3}r^2$.

   c. Find the ratio of r : h for A to be a minimum.

9. **(CA6.5 - N) (CA)** To introduce volumes of rotation: **(Oxford 9.6, p318)**

   a. Watch these videos to introduce the idea of "solids of revolution":

      i. Concept ⇒ https://www.youtube.com/watch?v=3oAjcLD34kc
      ii. Concept: First five minutes of https://www.youtube.com/watch?v=mQj0w8nVyc4
      iii. And finally here's how to do the math ⇒ https://www.youtube.com/watch?v=FGF0wP6THq4

   b. Try it yourself: To find the volume of the solid formed when the region bounded by the curve $g(x) = 6 - 2x$ and the x-axis between $x = 0$ and $x = 3$ is rotated 360° around the x-axis:
      i. Graph the function $g(x) = 6 - 2x$, between $x = 0$ and $x = 3$.
      ii. Shade in the region between $g(x)$ and the x-axis, between $x = 0$ and $x = 3$.
      iii. Perform the relevant integration to determine the volume of the 3D solid that would result from the rotation.
      iv. What 3D shape do you get?
      v. Determine the volume of this familiar 3D shape by using its volume formula.