Math SL PROBLEM SET 87

1. **(CA6.2 - R) (CI)** Find the equation of the derivatives of the following: **(Cirrito 19.3, p632)**

   a. $f(x) = \dfrac{\frac{1}{x} + \frac{1}{x^2}}{x - 1}$

   b. $y = \ln(xe^{7x})$

2. **(V4.4 - R) (CA)** The position vectors of the points A, B and C are given by OA = $\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$, OB = $\boldsymbol{i} + a\boldsymbol{j} - 2\boldsymbol{k}$ and OC = $b\boldsymbol{i} + 3\boldsymbol{j} + c\boldsymbol{k}$, where a, b and c are constants. Find:

   (Cirrito 12.3, p415)

   a. a if OA is perpendicular to OB
   b. b and c if O, A and C are collinear

3. **(F2.6 - R) (CI/CA)** The following questions deal with logarithmic functions and their domains.

   (Cirrito 5.3.4, p138)

   a. State the domain of $f(x) = \ln(x)$ as well as the domain of $g(x) = \ln(x - 2)$.
   b. Determine the domain of $y = \ln(x) - \ln(x - 2)$
   c. Determine the domain of $y = \ln\left(\frac{x}{x-2}\right)$
   d. Solve the equation $\ln(x) - \ln(x - 2) = -2$ (use a calculator to get approximate solutions - no graphic solutions)
   e. Solve the equation $\ln\left(\frac{x}{x-2}\right) = -2$ (use a calculator to get approximate solutions - no graphic solutions
   f. Now using graphing technology, graph $y = \ln(x) - \ln(x - 2)$ as well as $y = \ln\left(\frac{x}{x-2}\right)$ and explain your observations about the 2 graphs.
   g. Is $\ln(x) - \ln(x - 2) = \ln\left(\frac{x}{x-2}\right)$?

4. **(SP5.8 - R) (CA)** Ali is a BMW car salesman in Maadi (he didn't quite make it as a test driver!!) He has noticed that 10% of the cars he sells are "metallic grey" in colour. Twenty of his customers are selected at random and their new car orders are checked for colour preferences. Find the probability that:

   (Cirrito 16.3, p548)

   a. At least 5 cars are metallic grey in colour.
   b. At most 6 cars are metallic grey in colour.
   c. More than 5 are metallic grey in colour.

   In a sample of 100 customers records, find:

   d. The expected number of metallic grey car order.
   e. The standard deviation of metallic grey car orders.

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5. **(SP5.6 - R) (CA)** Events A and B are such that $P(A) = x$, $P(B) = 2x$ and $P(A \cup B) = ¾$.

(Cirrito 15.2, p510)

   a. Solve for x if the events A and B are mutually exclusive.
   b. Solve for x if the events are such that A $\subseteq$ B (NOTE: A is a subset of B)
   c. Solve for x if the events are independent.

6. **(T3.3 - R) (CA)** Triangle ABC has AB = 12 cm and BC = 9 cm and angle CAB = 0.55 radians. Side AC = x cm.

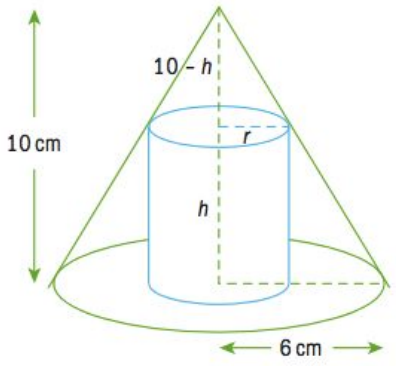
(Cirrito 9.5.4, p294)

   a. Explain why the equation $0.7 = \sin(x)$ has multiple solutions.
   b. Draw a diagram, labelling this information.
   c. Solve for x (side AC) using the sine law.
   d. Solve for x (side AC) using the cosine law.
   e. Explain why the solutions for the two previous questions are different.

7. **(CA6.3 - R) (CI)** A cylinder is inscribed in a cone with radius 6 centimetres and height 10 centimetres.

(Cirrito 21.4.3, p715)

   a. Show that an expression for r, the radius of the cylinder in terms of h, the height of the cylinder is $r = 6 - \frac{3h}{5}$.
   b. Show that an expression of the volume, V, of the cylinder in terms of h is $V(h) = \pi\left(36h - \frac{36}{5}h^2 + \frac{9}{25}h^3\right)$.
   c. Find $\frac{dV}{dh}$ and $\frac{d^2V}{dh^2}$.
   d. **(CA)** Hence find the radius and height of cylinder with maximum volume.



8. **(CA6.3 - R) (CI)** For the function $f(x) = \sin(x) + ½\sin(2x)$ on the domain of $0 \leq x \leq \pi$, find:

(Cirrito 20.2, p649)

   a. the x coordinates of the stationary point(s).
   b. Use the second derivative to confirm what type of stationary points of the function.
   c. Find the x coordinates of the inflection point(s)
   d. Sketch the graph based upon your information from Q8a, 8b and 8c.