Math SL PROBLEM SET 86

1. **(CA6.2 - R) (CI)** Find the equation of the derivatives of the following: **(Cirrito 19.3, p632)**

   a. $f(x) = \sqrt{\frac{2x+5}{7x-9}}$ b. $y = \tan(\cos(x))$

2. **(V4.4 - R) (CA)** The following pair of lines may OR may not intersect at a point. Find the point at which the lines intersect OR explain how you know they don't. **(Cirrito 12.7.3, p460)**

   L_1: $\boldsymbol{r} = (1,3,5) + \lambda(7,1,-3)$ and L_2: $\boldsymbol{r} = (4,6,7) + \mu(-1,0,2)$

3. **(SP5.7 - R) (CA)** The discrete random variable x has the probability function given by

$$P(x) = \begin{cases} \left(\frac{1}{4}\right)^{x-1} & x = 2,3,4,5,6 \\ k & x = 7 \\ 0 & x > 7 \end{cases}$$

   (Cirrito 16.2, p541)

   a. Find the value of k.
   b. Find $P(X = 5 \mid X < 7)$
   c. Find the expected value of x.

4. **(SP5.6 - R) (CA)** The airline, IB Airways, is known for their punctuality (and losing your luggage, but hey, there ya go!!!). The probability that a regularly scheduled flight leaves on time is P(D) = 0.83 and the probability that it arrives on time is P(A) = 0.92 and the probability that it arrives and leaves on time is $P(A \cap D) = 0.78$. Determine:

   (Oxford 3.4, p85)

   a. the probability that a flight arrives on time, given that it departed on time;
   b. the probability that a flight departs on time given that it arrived on time;
   c. whether or not the two events (on time arrival and departure) are independent

5. **(CA6.6 - E) (CI)** The velocity, v, in ms^{-1} of a particle moving in a straight line along the x-axis is given by the function $v(t) = \sin(\pi t)$. **(Cirrito 22.6, p764)**

   a. Determine when the particle is moving to the right and when it is moving to the left and when it is stopped.
   b. At any time it stops, determine whether it changes direction or not.
   c. Find the particle's displacement for the time interval $0 \leq t \leq 3$.
   d. Find the particle's total distance travelled for the time interval $0 \leq t \leq 3$.

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6. **(T2.3 - R) (CI)** Given a function, $f(x)$, and the knowledge that $f(-2) = 6$, determine the location of the image of point (-2,6) for the following transformations:

(Oxford 1.6, p21)

a. $T(x) = -2f(2x)$ b. $T(x) = f(\frac{1}{3}(x + 4))$ c. $T(x) = f(-\frac{1}{2}x + 1)$

7. **(V4.3 - R) (CI)** At 12:00 noon, Satellite A is orbiting and is currently above Cairo at a height of 120 km and a speed of 800 km/hr. The current direction vector of satellite A is given as (4,3). NOTE: the vector (1,0) means a displacement of 1 km due east and (0,1) means a displacement of 1 km due north. **(Cirrito 12.7.2, p452)**

a. Using Cairo as the origin (0,0), show that the position vector of satellite A at time t hours after noon is given as $r = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 640 \\ 480 \end{pmatrix}$

b. Find the position of the satellite at 14:30 pm.

c. Satellite B is heading towards Cairo with a velocity vector of (-300, -400) from a location of (600, 480) and also at a height of 120 km.

i. Find the speed of Satellite B.

ii. Is there a danger of collision?

8. **(CA6.3 - N) (CA)** A 10 foot post and a 25 foot post stand 30 feet apart and are perpendicular to the ground. Wires of lengths y and z run from the top of each pole and are attached by a single stake at a point on the ground between the two poles, as shown in the figure.

(Cirrito 21.4, p716)

a. Write down an expression for y in terms of x.

b. Write down an expression for z in terms of x.

c. Hence, write an expression for $L(x)$, the total length of the wire used for both poles.

d. Find $\frac{dL}{dx}$.

e. Hence, or otherwise, find the distance, x, the stake should be placed from the ten foot pole in order to minimize the amount of wire used. (use calculator for this)