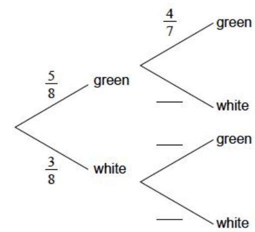
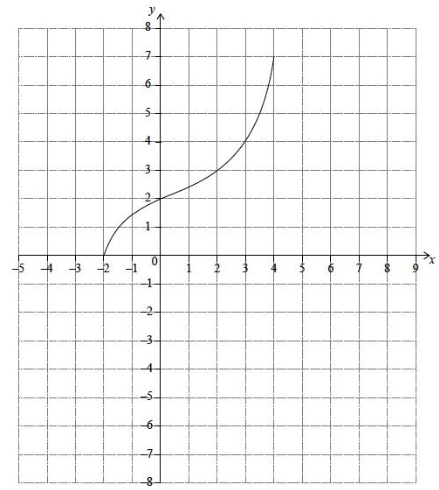
Math SL PROBLEM SET 84

1. (CI) A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.
   a. Complete the tree diagram.
   b. Find the probability that exactly one of the selected balls is green

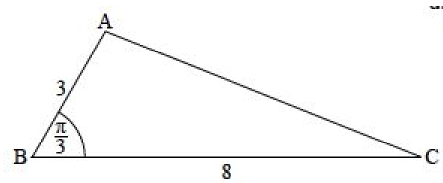


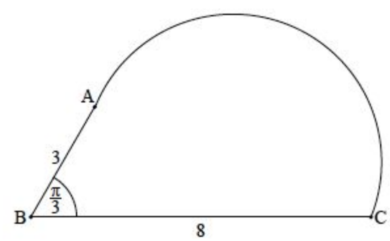
2. (CI) In an arithmetic sequence, the first term is 8 and the second term is 5.
   a. Find the common difference.
   b. Find the tenth term.
   c. Find the sum of the first ten terms.

3. (CI) The following diagram shows the graph of a function f, with domain $-2 \leq x \leq 4$. The points $(-2, 0)$ and $(4, 7)$ lie on the graph of f .
   a. Write down the range of f.
   b. Write down
      i. $f(2)$;
      ii. $f^{-1}(2)$.
   c. On the grid, sketch the graph of f^{-1}.



4. (CI) The following diagram shows ΔABC, with AB = 3 cm , BC = 8 cm , and $\angle$ABC = $\frac{\pi}{3}$.
   a. Show that AC = 7 cm .
   b. The shape in the second diagram is formed by adding a semicircle with diameter [AC] to the triangle. Find the exact perimeter of this shape.

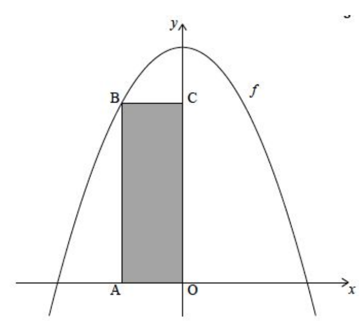




5. (CI) Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for x $\in \boldsymbol{R}$, where b is a constant.
   a. Find $(g \circ f)(x)$.
   b. Given that $\lim_{x \to +\infty} (g o f)(x) = -3$, find the value of b .

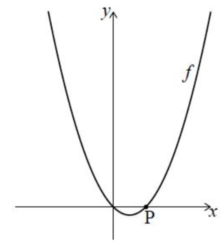
6. (CI) Consider $f(x) = \log_k (6x - 3x^2)$, for $0 < x < 2$, where $k > 0$. The equation $f(x) = 2$ has exactly one solution. Find the value of k .

7. (CI) Let $f(x) = 15 - x^2$, for x $\in \boldsymbol{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f , and C is on the y-axis. Find the x-coordinate of A that gives the maximum area of OABC.



Math SL PROBLEM SET 84

8. (CI) Let $f(x) = x^2 - x$, for $x \in \boldsymbol{R}$. The following diagram shows part of the graph of f . The graph of f crosses the x-axis at the origin and at the point P (1 , 0) .

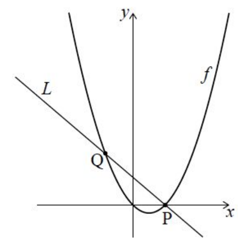


   a. Show that $f'(1) = 1$.

   The line L is the normal to the graph of f at P.

   b. Find the equation of L in the form $y = ax + b$.

   The line L intersects the graph of f at another point Q, as shown in the second diagram.



   c. Find the x-coordinate of Q.
   d. Find the area of the region enclosed by the graph of f and the line L .

9. (CI) A line L passes through points A (-3 , 4 , 2) and B (-1 , 3 , 3) .

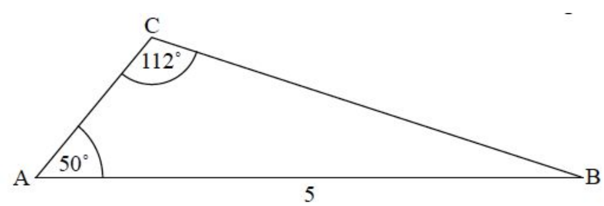
   a. (i) Show that $\vec{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

      (ii) Find a vector equation for L .

   The line L also passes through the point C (3 , 1 , p) .

   b. Find the value of p .
   c. The point D has coordinates (q^2 , 0 , q) . Given that vector DC is perpendicular to L , find the possible values of q .

10. (CA) The following diagram shows a triangle ABC where AB = 5 cm , ∠CAB = 50° and ∠ACB =112°



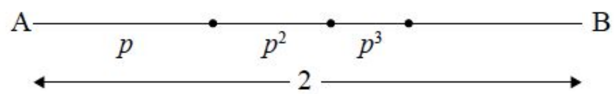
   a. Find BC.
   b. Find the area of triangle ABC.

11. (CA) Let $f(x) = \frac{6x^2 - 4}{e^x}$, for $0 \le x \le 7$.

   a. Find the x-intercept of the graph of f .
   b. The graph of f has a maximum at the point A. Write down the coordinates of A.
   c. Sketch the graph of f .

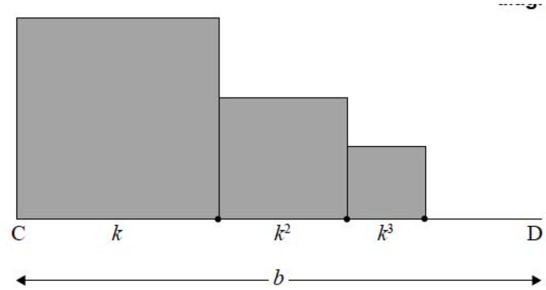
Math SL PROBLEM SET 84

12. (CI) The following diagram shows [AB], with length 2 cm. The line is divided into an infinite number of line segments. The diagram shows the first three segments. The length of the line segments are p cm , p^2 cm , p^3 cm , … , where $0 < p < 1$.



   a. Show that p = ⅔.

   The following diagram shows [CD], with length b cm , where $b > 1$. Squares with side lengths k cm , k^2 cm , k^3 cm , … , where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.



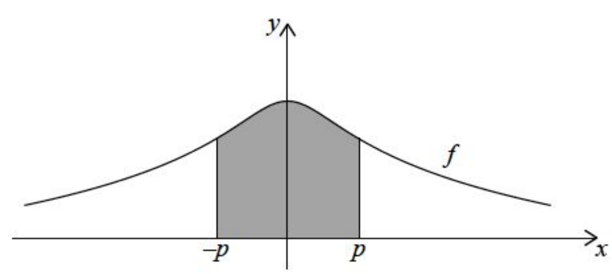
   b. The total sum of the areas of all the squares is 9/16. Find the value of b .

13. (CA) A discrete random variable X has the following probability distribution.

   a. Find the value of k .
   b. Write down $\mathrm{P}(X = 2)$.
   c. Find $\mathrm{P}(X = 2 \mid X > 0)$.

X	0	1	2	3
$\mathrm{P}(X=x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

14. (CA) Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \boldsymbol{R}$. The graph of f passes through the point $(p , 4)$, where $p > 0$.



   a. Find the value of p .
   b. The following diagram shows part of the graph of f. The region enclosed by the graph of f , the x-axis and the lines $x = -p$ and $x = p$ is shaded.
   Find the area enclosed by the x-axis, $f(x)$, and the lines $x = p$ and $x = -p$.

15. (CA) In the expansion of $ax^3(2 + ax)^{11}$, the coefficient of the term in x^5 is 11880. Find the value of a .

Math SL PROBLEM SET 84

16. (CA) Let $\vec{AB} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

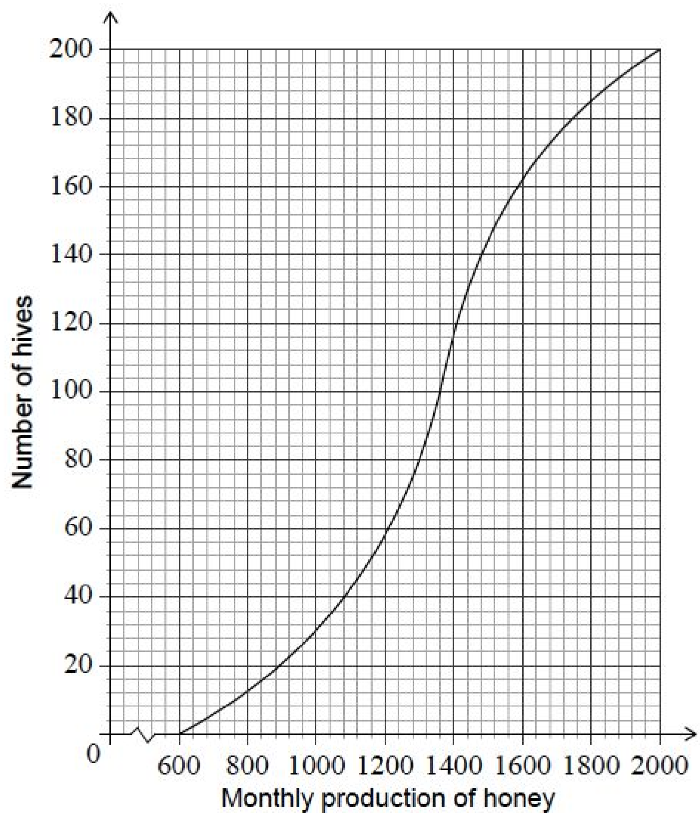
  a. Find |AB|.
  b. Find ∠BAC if $\vec{AC} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

17. Adam is a beekeeper who collected data about monthly honey production in his bee hives. The data for six of his hives is shown in the following table. The relationship between the variables is modelled by the regression line with equation $P = aN + b$.

Number of bees (N)	190	220	250	285	305	320
Monthly honey production in grams (P)	900	1100	1200	1500	1700	1800

  a. Write down the value of a and of b.
  b. Use this regression line to estimate the monthly honey production from a hive that has 270 bees.

Adam has 200 hives in total. He collects data on the monthly honey production of all the hives. This data is shown in the following cumulative frequency graph.



Math SL PROBLEM SET 84

Adam's hives are labelled as low, regular or high production, as defined in the following table.

Type of hive	low	regular	high
Monthly honey production in grams (P)	$P \leq 1080$	$1080 < P \leq k$	$P > k$

c. Write down the number of low production hives.

Adam knows that 128 of his hives have a regular production.

d. Find
   i. the value of k ;
   ii. the number of hives that have a high production.

e. Adam decides to increase the number of bees in each low production hive. Research suggests that there is a probability of 0.75 that a low production hive becomes a regular production hive. Calculate the probability that 30 low production hives become regular production hives.

18. (CA) Note: In this question, distance is in metres and time is in seconds. A particle P moves in a straight line for five seconds. Its acceleration at time t is $a(t) = 3t^2 - 14t + 8$, for $0 \leq t \leq 5$.

a. Write down the values of t when $a(t) = 0$.
b. Hence or otherwise, find all possible values of t for which the velocity of P is decreasing.

When $t = 0$, the velocity of P is 3 ms^{-1} .

c. Find an expression for the velocity of P at time t .
d. Find the total distance travelled by P when its velocity is increasing.

19. (CA) Note: In this question, distance is in millimetres. Let $f(x) = x + a\sin\left(x - \frac{\pi}{2}\right) + a$, for $x \geq 0$.

a. Show that $f(2\pi) = 2\pi$.

The graph of f passes through the origin. Let P_k be any point on the graph of f with x-coordinate $2k\pi$, where $k \in \mathbb{N}$. A straight line L passes through all the points P_k .

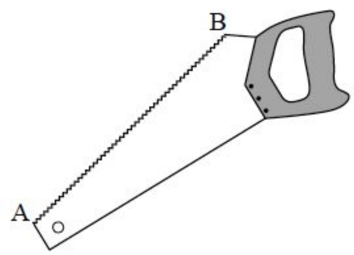
Math SL PROBLEM SET 84

b. (i) Find the coordinates of P_0 and of P_1 .
   (ii) Find the equation of L .

c. Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π .

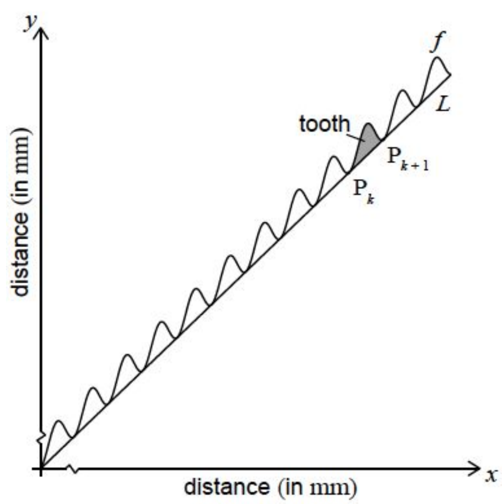
Diagram 1 shows a saw. The length of the toothed edge is the distance AB.

Diagram 1



The toothed edge of the saw can be modelled using the graph of f and the line L . Diagram 2 represents this model.

Diagram 2



The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of f and the line L , between P_k and P_{k+1} .

d. A saw has a toothed edge which is 300 mm long. Find the number of complete teeth on this saw.

(November 2017)