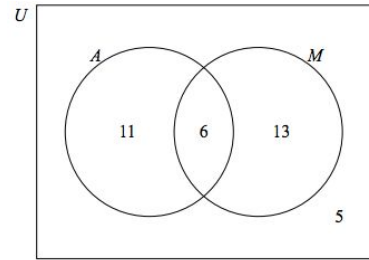


Math SL PROBLEM SET 83

1. **(CA)** In a group of 35 students, some take art class (A) and some take music class (M). 5 of these students do not take either class. This information is shown in the following Venn diagram.

- a. Write down the number of students in the group who take art class.
b. One student from the group is chosen at random. Find the probability that
- the student does not take art class;
 - the student takes either art class or music class, but not both



2. **(CA)** This table shows the hand lengths and the heights of five athletes on a sports team. The relationship between x and y can be modelled by the regression line with equation $y = ax + b$.

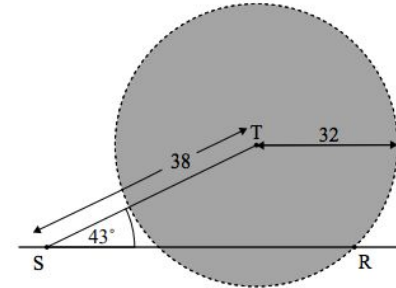
Hand length (x cm)	21.0	21.9	21.0	20.3	20.8
Height (y cm)	178.3	185.0	177.1	169.0	174.6

- a. (i) Find the value of a and of b .
(ii) Write down the correlation coefficient.
b. Another athlete on this sports team has a hand length of 21.5 cm. Use the regression equation to estimate the height of this athlete.
3. **(CA)** Let $f(x) = \frac{6x-1}{2x+3}, x \neq -\frac{3}{2}$.
- a. For the graph of f ,
- find the y -intercept;
 - find the equation of the vertical asymptote;
 - find the equation of the horizontal asymptote.
- b. Hence or otherwise, write down $\lim_{x \rightarrow \infty} \frac{6x-1}{2x+3}$.
4. **(CA)** A particle moves along a straight line so that its velocity, $v \text{ ms}^{-1}$, after t seconds is given by $v(t) = 1.4^t - 2.7$ for $0 \leq t \leq 5$.
- Find when the particle is at rest.
 - Find the acceleration of the particle when $t = 2$.
 - Find the total distance travelled by the particle.
5. **(CA)** The sum of an infinite geometric sequence is 33.25. The second term of the sequence is 7.98. Find the possible values of r .

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6. **(CA)** Consider the expansion of $\left(2x^4 + \frac{x^2}{k}\right)^{12}$; $k \neq 0$. The coefficient of the term in x^{40} is five times the coefficient of the term in x^{38} . Find the value of k .

7. **(CA)** A communication tower, T, produces a signal that can reach cellular phones within a radius of 32 km. A straight road passes through the area covered by the tower's signal. The following diagram shows a line representing the road and a circle representing the area covered by the tower's signal. Point R is on the circumference of the circle and points S and R are on the road. Point S is 38 km from the tower and $\angle RST = 43^\circ$.

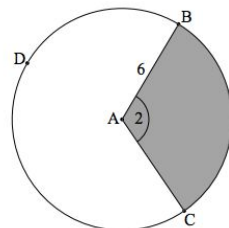


- Let $SR = x$. Use the cosine rule to show that $x^2 - (76 \cos 43^\circ)x + 420 = 0$.
 - Hence or otherwise, find the total distance along the road where the signal from the tower can reach cellular phones.
8. **(CA - ER)** Consider the points $A(-3, 4, 2)$ and $B(8, -1, 5)$.

- (i) Find vector \overrightarrow{AB} .
- (ii) Find the $|\overrightarrow{AB}|$.

A line L has vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. The point $C(5, y, 1)$ lies on line L.

- (i) Find the value of y .
 - (ii) Show that vector $\overrightarrow{AC} = 8\mathbf{i} - 10\mathbf{j} - \mathbf{k}$.
- Find the angle between vector \overrightarrow{AB} and vector \overrightarrow{AC} .
 - Find the area of triangle ABC.
9. **(CI)** The following diagram shows a circle with centre A and radius 6 cm. The points B, C, and D lie on the circle, and $\widehat{BAC} = 2$ radians.



- Find the area of the shaded sector.
 - Find the perimeter of the non-shaded sector ABDC.
10. **(CI)** Let $b = \log_2 a$, where $a > 0$. Write down each of the following expressions in terms of b .

- $\log_2 a$
- $\log_2 8a$
- $\log_8 a$

Math SL PROBLEM SET 83

11. **(CI)** Two functions, f and g , are defined in the following table.

- Write down the value of $f(1)$.
- Find the value of $(g \circ f)(1)$.
- Find the value of $g^{-1}(-2)$.

x	-2	1	3	6
$f(x)$	6	3	1	-2
$g(x)$	-7	-2	5	9

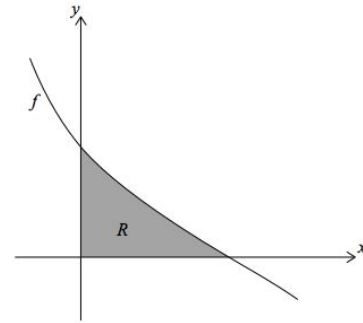
12. **(CI)** In an arithmetic sequence, $u_1 = -5$ and $d = 3$.

- Find u_8 .
- Find the value of n for which $u_n = 67$.

$$\mathbf{a} = \begin{pmatrix} 3 \\ 2p \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} p+1 \\ 8 \end{pmatrix}.$$

13. **(CI)** Consider the vectors \mathbf{a} and \mathbf{b} are parallel. Find the possible values of p for which \mathbf{a} and \mathbf{b} are parallel.

14. **(CI)** Let $f(x) = \frac{6-2x}{\sqrt{16+6x-x^2}}$. The following diagram shows part of the graph of f . The region R is enclosed by the graph of f , the x -axis, and the y -axis. Find the area of R .



15. **(CI)** Given that $\sin(x) = \frac{1}{3}$, where $0 \leq x \leq \frac{\pi}{2}$, find the value of $\cos(4x)$.

16. **(CI - ER)** Let $f(x) = x^2 - 4x - 5$.

- Find the x -intercepts of the graph of f .
- Find the equation of the axis of symmetry of the graph of f .
- The function can be written in the form $f(x) = (x - h)^2 + k$.
 - Write down the value of h .
 - Find the value of k .

The graph of a second function, g , is obtained by a reflection of the graph of f in the y -axis,

followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

- Find the coordinates of the vertex of the graph of g .

Math SL PROBLEM SET 83

17. **(CI - ER)** A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

- a. Find the probability, in terms of n , that the game will end on her
 - i. first draw;
 - ii. second draw.

- b. Let $n = 5$. Find the probability that the game will end on her
 - i. (i) third draw;
 - ii. (ii) fourth draw.

Hayley plays the game when $n = 5$. She pays \$20 to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let M be the amount of money that she earns back playing the game. This information is shown in the following table.

Number of draws	1	2	3	4
Money earned back (\$$M$)	0	20	$8k$	$12k$

- c. Find the value of k so that this is a fair game.

18. **(CI - ER)** Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram. The graph of f crosses the y -axis at the point P . The line L is tangent to the graph of f at P .

- a. Find the coordinates of P .
- b. (i) Find $f'(x)$.
(ii) Hence, find the equation of L in terms of a . [6]

The graph of f has a local minimum at the point Q . The line L passes through Q .

- c. Find the value of a .

