Math SL PROBLEM SET 80

1. **(CA6.2 - N) (CI)** Determine the equation of the line that is tangent to the following curves at the specified points. Confirm using your TI-84.

(Cirrito 20.1, p643)

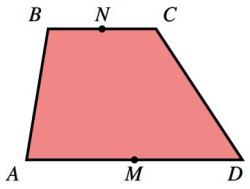
   a. $g(x) = \frac{e^x}{x}$ at the point where $x = ½$.
   b. $h(x) = xe^{-2x} + 2$ at the point where $h(x)$ crosses the y-axis.
   c. $x^2y - y = x^2 - 4$ at the point where the function crosses the x-axis.

2. (CA6.4 - N) (CI) Evaluate the following integrals. Confirm using your TI-84.

(Oxford 9F, p302)

   a. $\int_0^1 xe^{x^2}\,dx$ b. $\int_0^4 sin(\sqrt{x})\,\frac{dx}{\sqrt{x}}$ c. $\int \frac{6x^2}{x^3+1}dx$

3. **(V4.1 - R) (CI)** ABCD is a trapezium with BC parallel to AD. M is the midpoint of AD and N is the midpoint of BC. Given that vector AB $= 2\boldsymbol{a}$, vector BC $= 2\boldsymbol{z}$ and vector AD $= 6\boldsymbol{z}$, express vector MN in terms of $\boldsymbol{z}$ and $\boldsymbol{a}$. **(Oxford 12.4, p424)**



4. **(T3.5 - R) (CI)** Solve for θ in the equation $2\sin^2\theta = 3\cos\theta$, where $0 \leq \theta \leq 2\pi$. Confirm using your TI-84

(Cirrito 10.4, p351)

5. **(CA6.5 - E) (CI)** Given $f(x) = \sin(x)$ and $g(x) = \sqrt{3}\cos(x)$ and the domain of $0 < x < \frac{3\pi}{2}$. Confirm using your TI-84.

(Cirrito 22.5, p760)

   a. Find the point(s) at which $f(x)$ and $g(x)$ intersect in the given domain.
   b. Find the area of the region bounded by f and g.

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6. **(CA6.3 - R) (CI)** Given $f(x) = \frac{sin(x)}{e^x}$ on $0 \leq x \leq 2\pi$. Confirm all answer using your TI-84.

(Cirrito 20.2, p669)

   a. Determine the equations for $\frac{df}{dx}$ and for $\frac{d^2f}{dx^2}$.
   b. Hence, determine where $f(x)$ has stationary points and inflection points.
   c. Classify the stationary points, justifying your conclusions using either the first or second derivatives.
   d. Sketch $f(x)$.

7. **(V4.2, 4.3 - R) (CA)** Points A, B, and C have position vectors $4\boldsymbol{i} + 2\boldsymbol{j}$, $\boldsymbol{i} - 3\boldsymbol{j}$ and $-5\boldsymbol{i} - 5\boldsymbol{j}$. Let D be a point on the x-axis such that ABCD forms a parallelogram.

(Cirrito 12.7, p444)

   a. Find vector BC.
   b. Find the position vector of D.
   c. Find the angle between vector BD and vector AC.

   The line L_1 passes through A and is parallel to $\boldsymbol{i} + 4\boldsymbol{j}$. The line L_2 passes through B and is parallel to $2\boldsymbol{i} + 7\boldsymbol{j}$. A vector equation of L_1 is $\boldsymbol{r} = (4\boldsymbol{i} + 2\boldsymbol{j}) + \lambda(\boldsymbol{i} + 4\boldsymbol{j})$.

   d. Write down a vector equation of L_2 in the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{q}$.
   e. The lines L_1 and L_2 intersect at the point P. Find the position vector of P.

8. **(CA6.3 - E) (CA)** The diagram below shows a sketch of the graph of the function $y = \sin(e^x)$ where $-1 < x < 2$, and x is in **radians**. The graph cuts the y-axis at A, and the x-axis at C and D. It has a maximum point at B.

(Cirrito 20.2, p649)

   a. Find the coordinates of A.
   b. The coordinates of C may be written as $(\ln k, 0)$. Find the **exact** value of k.
   c. Write down the y-coordinate of B. (Hint: CA??)
   d. Find $\frac{dy}{dx}$.
   e. Hence, show that at B, $x = \ln \frac{\pi}{2}$.
   f. Write down the integral which represents the shaded area.
   g. Evaluate this integral. (Hint: CA???)