Math SL PROBLEM SET 78

1. **(CA6.2 - R) (CI)** Differentiate (and simplify) the following:

(Cirrito 19.3, p618)

a. $y = \frac{1}{x^2}\ln(x)$ (b) $y = \sqrt{1 + sin^2(x)}$ (c) $y = \frac{sin(x)}{1 - cos(x)}$

2. **(T3.2 - R) (CI)** Evaluate the following: **(Cirrito 9.1.2, p273)**

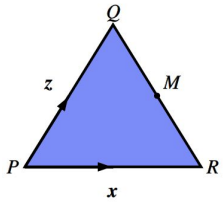
a. i.) $\sin\left(\frac{3\pi}{4}\right)$ ii.) $\cos\left(-\frac{2\pi}{3}\right)$ iii.) $\tan\left(\frac{-5\pi}{6}\right)$

Use the domain of $-2\pi \leq x \leq 2\pi$ to answer Q2b

b. i.) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ ii.) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ iii.) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

3. **(V4.1 - R) (CI)** Triangle PQR is shown below where vector PQ $= \boldsymbol{z}$, vector PR $= \boldsymbol{x}$ and M is the midpoint of QR. Express the following vectors in terms of $\boldsymbol{z}$ and $\boldsymbol{x}$. **(Oxford, 12.4, p424)**

a. vector QR
b. vector QM
c. vector PM



4. **(T3.5 - R) (CI)** The population (in thousands) of a species of butterfly in a nature sanctuary is modeled by the function $P(t) = 3 + 2\sin\left(\frac{3\pi t}{8}\right)$, $0 \leq t \leq 12$, where t is time in weeks after the scientists first started making observations.

(Cirrito 10.5, p365)

a. The function $P(t)$ represents a transformation of the parent function, $y(t) = \sin(t)$. How has the function $y(t) = \sin(t)$ been transformed to create the equation for $P(t)$?
b. What was the initial population of the butterflies?
c. What are the maximum and minimum numbers of the butterfly population?
d. What is the period of this function?
e. Sketch the function, $y = P(t)$.
f. When is the first time that the population reaches 4000?
g. At what rate is the butterfly population changing at $t = 2$.

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5. **(CA6.4 - N) (CI)** Evaluate the following integrals:

(Oxford 9F, p302)

a. $\int_0^1 x^2(x^3+2)^3\,dx$ b. $\int \sqrt{4x-5}\,dx$ c. $\int \frac{4}{5-2x}dx$

6. **(CA6.3 - E) (CI)** Given the function $h(x) = \frac{x-2}{(x-1)^2}$, $x \neq 1$. **(Cirrito 20.2, p649)**

a. Determine the equation(s) of the horizontal and vertical asymptote(s).
b. Determine the x- and y-intercepts (if possible).
c. Find the derivative of $h(x)$, writing your answer in the form of $\frac{a-x}{(x-1)^n}$, where a and n are constants to be determined (by you of course ……)
d. Show that the second derivative is $h``(x) = \frac{2x-8}{(x-1)^4}$.
e. Find the coordinates of the extrema(s) and inflection point(s).

7. **(CA6.5 - E) (CI)** Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = x$.

(Cirrito 22.5, p760)

8. (**SP5.7 - E**) (**CA**) Mr S has set up the following game to raise some money for the Senior class. Three dice are thrown. If a 1 or a 6 is rolled somewhere on these three dice, you will be paid \$1, but if neither is rolled, you will pay \$5. **(Cirrito 16.2, p535)**

a. What is the probability that you will win \$1? (HINT: consider cases, binomials or trees)
b. Complete the following probability distribution table for the random variable X, which represents the number of dollars won in the game.

x	-5	1
$P(X=x)$		

c. How much would you expect to gain (or lose) in (i) one game? (ii) nine games?
d. Mr D would like to make the game fair. How much should you now be paid if you win?