Math SL PROBLEM SET 75

1. **(CA6.1, CA6.2 - R) (CI)** Determine the slope of the functions below at the specified points:
(Cirrito 20.1, p643)

   a. $f(x) = xe^{x^2+1}$ at $x = 0$.
   b. $g(x) = \ln(x^2 + 4)$ at the point where the curve crosses the x-axis.
   c. $h(x) = x\tan(x)$ at $x = \frac{\pi}{4}$

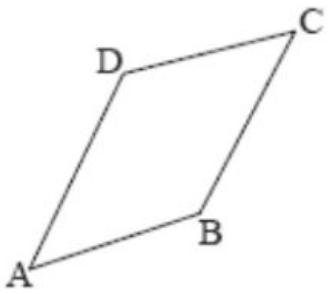
2. **(CA6.1 - R) (CI)** Find the x-coordinate(s) on the graph of $f(x) = 2x^3 - 7x + 1$ at which the tangent line is parallel to $5x - y = 2$ **(Cirrito 20.1, p643)**

3. **(F2.6 - R) (CI)** Solve each of the following equations for x, giving exact values in terms of natural logs (ln) or in terms of e (if necessary) **(Cirrito 7.3, p217; Cirrito 7.4, p221)**

   a. Solve $3^x = 6$
   b. Solve $\ln(3x + 1) - \ln(4 - x) = \ln 4$

4. **(V4.2 - R) (CI)** The following diagram shows quadrilateral ABCD, with
$\overrightarrow{AD} = \overrightarrow{BC}$, $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$
(Cirrito 12.4, p423)

   a. Find $\overrightarrow{BC}$
   b. Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$
   c. Show that vectors $\overrightarrow{BD}$ and $\overrightarrow{CA}$ are perpendicular.
   d. Determine the cosine ratio of the angle at A.



5. **(A1.1 - R) (CA)** The sum of the first five terms of a geometric series is 3798, and the sum to infinity is 4374. **(Cirrito 8.2.4, p263)**

   a. Find the sum of the first seven terms.
   b. Find the value of n such that S_n first exceeds 4200.

Math SL PROBLEM SET 75

6. **(C6.5 - E) (CI)** Given the functions $f(x) = x^2 + 1$ and $g(x) = 3 - x$. **(Cirrito 22.5.8, p755)**

   a. Sketch a graph showing the region bounded by these two functions.
   b. Write down an expression that gives the area of the region.
   c. Hence, or otherwise, find the area of this region.

7. **(V4.2 - R) (CA)** The position vectors of the points A, B and C are $\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, $2\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$ and $3\boldsymbol{i} + 4\boldsymbol{k}$ respectively. Find **(Cirrito 12.5, p429)**

   a. the angle BAC to the nearest degree;
   b. the area of triangle ABC.

8. **(CA6.3 - E) (CA)** A cylindrical tin with no lid is to be made such that its total surface area measures 100 cm^2.

   (Cirrito 21.4, p716)

   a. Given that the radius of the base of the tin is r cm, show that its volume, V cm^3, is given by $V(r) = \frac{1}{2}(100r - \pi r^3)$
   b. Determine the value of r that will give the greatest volume. Use the second derivative test to confirm your value.