Math SL PROBLEM SET 71

1. (**C6.2 - R**) (**CI**) The curve with the equation $y = Ax + B + \frac{C}{x}$ has a minimum at $P(1,4)$ and a maximum at $Q(-1,0)$. Find the value of each of the coefficients: A, B and C.

   (Cirrito 20.2, p649)

2. **(SP5.5 - R)** (**CA**) A box contains 7 blue marbles and 5 yellow marbles. Mark takes out three marbles, one after another, without replacement. Determine

   (Oxford 3.5, p89)

   a. the probability that Mark takes out three blue marbles;
   b. the probability that Mark takes out exactly 2 yellow marbles.
   c. Given that Mark has at least one blue marble, find the probability that Mark has taken out exactly 2 yellow marbles.

3. (**C6.2 - R**) (**CI**) The normal to the curve $y = x^{\frac{1}{2}} + x^{\frac{1}{3}}$ at the point (1,2) meets the axes at $(a,0)$ and $(0,b)$. Find the values of a and b.

   (Cirrito 20.2, p649)

4. (**C6.3 - N**) (**CI**) Find the equation of both the tangent and the normal to the curve $h(x) = x\tan(x)$ at the point where $x = \frac{\pi}{4}$.

   (Cirrito 20.2, p649)

5. (**C6.4 - N**) (**CI**) Evaluate the following indefinite integrals:

   (Cirrito 22.1, p723)

   a. $\int e^{-4x}\,dx$ b. $\int 2sin(3x)\,dx$ c. $\int (x^4 + 3x^2 - 4 - \frac{2}{x})\,dx$ d. $\int (2\sqrt{x} - \frac{3}{2\sqrt{x}})\,dx$

6. (**C6.5 - N**) (**CI**) Evaluate the following definite integrals and verify using technology:

   (Cirrito 22.4, p740)

   a. $\int_1^3 3x^2\,dx$ b. $\int_{-1}^2 4x^3\,dx$ c. $\int_0^{\frac{3\pi}{4}} cos(x)\,dx$ d. $\int_1^3 \frac{a^5+1}{a^2}\,da$ (HINT: Simplify first)

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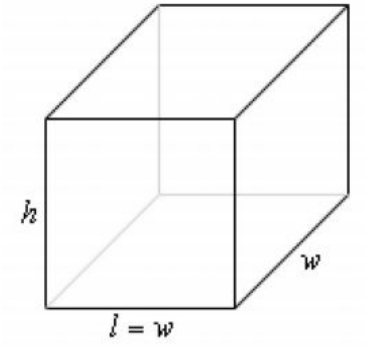
7. **(C6.3 - N)** **(CA)** We want to construct a box with a square base and we only have 10 m^2 of material to use in construction of the box. Assuming that all the material is used in the construction process, you will determine the maximum volume that the box can have.

(Cirrito 21.4, p702)



   a. Show that the volume formula is $V(w) = \frac{1}{2}(5w - w^3)$, where w is the width of the box.
   a. Find the value of w that optimizes the volume of the box.
   b. Use the **second derivative test** to verify that your value for w does give a **maximum** volume.

8. **(C6.2 - R; SP5.9 - N)** **(CA/CI)** A very important function in statistics is the equation for the standard normal curve, given by $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, where the mean is 0 and the standard deviation is 1.

   a. (CI) Use calculus to find the x-coordinates of any stationary points and any inflection points.
   b. (CI) Find what happens when $x \to \infty$ and when $x \to -\infty$. Give the equation(s) of any asymptotes.
   c. (CA) Graph the function on your calculator and record your window settings. Explain why these window settings "make sense".
   d. (CA) Use your graph to evaluate the following integrals:
      i. $\int_0^1 f(x)dx$
      ii. $\int_{-1}^0 f(x)dx$
      iii. $\int_0^2 f(x)dx$
      iv. $\int_{-3}^3 f(x)dx$