Math SL PROBLEM SET 104

Section A (Skills/Concepts Consolidation)

[Maximum mark: 6]

The sum of the first three terms of a geometric sequence is 81.3, and the sum of the infinite sequence is 300. Find the common ratio.

[Maximum mark: 6]

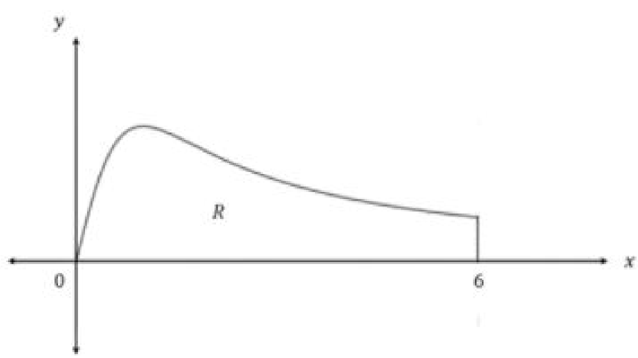
Let $f(x) = e^{-3x}$.

(a) Write down $f'(x)$, $f''(x)$ and $f'''(x)$. [3]

(b) Find an expression for $f^{(n)}(x)$. [3]

[Maximum mark: 6]

The following diagram shows the graph of $f(x) = \frac{4x}{x^2+1}$, for $0 \le x \le 6$, and the line $x = 6$.



Let R be the region enclosed by the graph of f, the x-axis and the line $x = 6$.

Find the area of R.

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[Maximum mark: 7]

In the expansion of $x(2x+1)^n$, the coefficient of the term in x^3 is $20n$, where $n \in \mathbb{Z}^+$.
Find n.

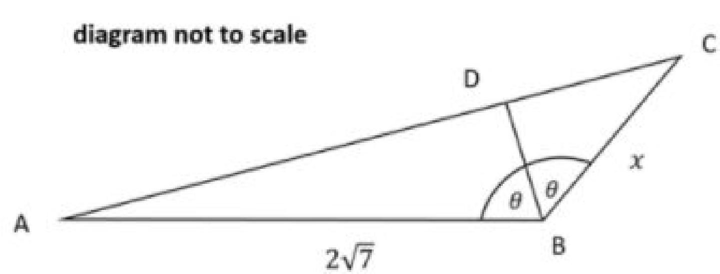
[Maximum mark: 7]

The time taken for a student to finish a task is normally distributed with a mean μ and standard deviation σ. It is found that 6% of students take less than 7 minutes to complete the task and 75% take less than 22 minutes.

Find the value of μ and of σ.

[Maximum mark: 7]

The following diagram shows triangle ABC. Point D lies on AC so that DB bisects $\mathrm{A\hat{B}C}$.



$\mathrm{AB} = 2\sqrt{7}\ \mathrm{cm}$, $\mathrm{BC} = x\ \mathrm{cm}$, and $\mathrm{D\hat{B}C} = \theta$, where $\sin\theta = \frac{3}{4}$

The area of the triangle ABC is $3\ \mathrm{cm}^2$. Find the value of x in the form of $\frac{a}{b}$ where a and b are positive integers.

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[Maximum mark: 7]

Consider the functions $f(x) = 2(x-h)^2 + k$ and $g(x) = (x-h)^2 + k$.

The vertex of f is at $(m, -m^2)$ and the vertex of g is at $(-m, -m)$, where $0 < m < 1$.

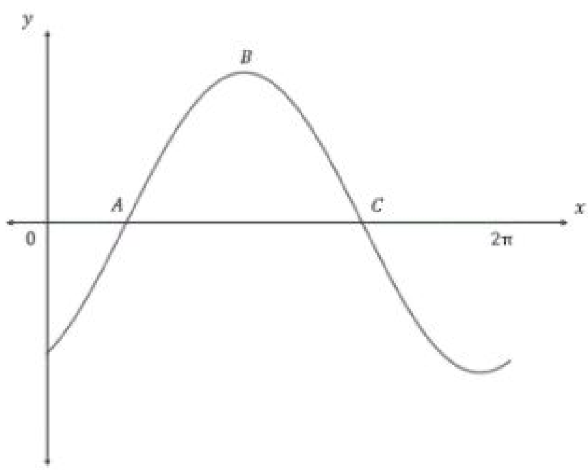
The graphs of f and g intersect at exactly one point. Find the value of m.

Section B (Skills/Concepts Practice)

[Maximum mark: 15]

Let $f(x) = \sin x - \sqrt{3}\cos x,\ 0 \le x \le 2\pi$.

The following diagram shows the graph of f.



The curve crosses the x-axis at A and C and has a maximum at point B.

(a) Find the exact coordinates of A and of C. [8]

(b) Find $f'(x)$. [2]

(c) Find the coordinates of B. [5]

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[Maximum mark: 15]

Consider the points A (3, 2, -5) and B (-3, 6, -5).

(a) Find $\overrightarrow{AB}$. [2]

Let C be a point such that $\overrightarrow{AC} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$.

(b) Find the coordinates of C. [2]

The line L passes through B and is parallel to $\overrightarrow{AC}$.

(c) Write down a vector equation for L. [2]

(d) Given that $|\overrightarrow{AB}| = k|\overrightarrow{AC}|$, find k. [3]

(e) The point D lies on L such that $|\overrightarrow{AB}| = |\overrightarrow{BD}|$. Find the possible coordinates for D. [6]

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[Maximum mark: 15]

The following table shows a probability distribution for the random variable X, where $E(X) = 0.8$.

x	0	1	2
$P(X = x)$	p	$\frac{8}{15}$	q

(a) (i) Find q.

(ii) Find p. [4]

A bag contains black and red balls, with at least two of each colour. Two balls are drawn from the bag without replacement. The number of red balls drawn is given by the random variable X.

(b) (i) Write down the probability of drawing two red balls.

(ii) Explain why the probability of drawing two black balls is $\frac{1}{3}$.

(iii) The bag contains a total of ten balls of which k are black. Find k. [5]

A game is played in which two balls are drawn from the bag of ten balls, without replacement. A player wins a prize if two black balls are drawn.

(c) Olivia plays the game twelve times. Find the probability that she wins exactly three prizes. [2]

(d) William plays the game until he wins three prizes. Find the probability that he wins his third prize on his tenth attempt. [4]