Math SL PROBLEM SET 103

Section A (Skills/Concepts Consolidation)

[Maximum mark: 6]

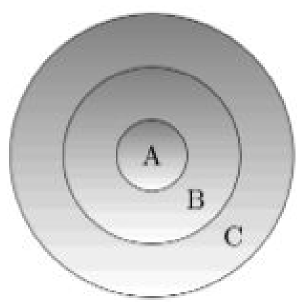
The following box-and-whisker plot shows the number of tweets sent by people in a coffee shop on a particular day.

(a) Find the value of the interquartile range. [2]

(b) One person sent k tweets, where $k > 7$. Given that k is an outlier, find the least value of k. [4]

[Maximum mark: 7]

The following diagram shows an archery target which is divided into three regions A, B and C.



A contest consists of an archer shooting one arrow at the target. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\frac{1}{24}$	$\frac{4}{24}$	$\frac{7}{24}$

(a) Find the probability that the arrow does **not** hit the target. [3]

The archer scores points as shown in the following table.

Region	A	B	C	Outside Target
Points	10	6	k	-4

(b) Given that the contest is fair, find the value of k. [4]

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[Maximum mark: 7]

Let $f(x) = ax^2 - 24x + c$. A horizontal line, L, intersects the graph of f at $x = 1$ and $x = 7$.

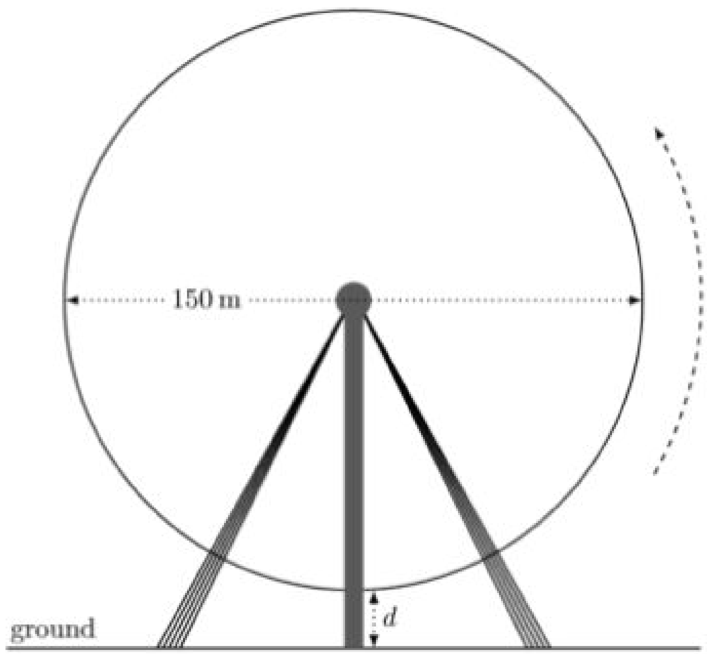
(a) (i) The equation of the axis of symmetry is $x = h$. Find h.

(ii) Hence, show that $a = 3$. [4]

(b) The equation of L is $y = 6$. Find the value of c. [3]

[Maximum mark: 8]

The Singapore Flyer is a giant Ferris wheel in Singapore with diameter of 150 metres. The bottom of the wheel is d metres above the ground. A seat starts at the bottom of the wheel.



The wheel rotates at a constant speed and completes one revolution in 32 minutes.

(a) After 16 minutes, the seat is 165 metres above the ground. Find d. [2]

After t minutes, the height of the seat above the ground is given by $h(t) = 90 + a\cos\left(\frac{\pi}{16}t\right)$, for $0 \le t \le 64$.

(b) Find the value of a. [3]

(c) Find when the seat is 60 metres above the ground for the third time. [3]

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[Maximum mark: 6]

Consider the expansion of $\left(3x+\frac{k}{x}\right)^8$, where $k>0$. The coefficient of the term in x^4 is equal to the coefficient of the term in x^6. Find k.

[Maximum mark: 7]

The expression $8\sin x\cos x$ can be written in the form $p\sin qx$.

(a) Find the value of p and the value of q. [3]

(b) Hence or otherwise, solve the equation $8\sin x\cos x=2\sqrt{3}$, for $\frac{\pi}{4}\le x\le\frac{\pi}{2}$. [4]

[Maximum mark: 7]

Let $f(x)=\frac{9-12x}{cx-20}$, for $x\neq\frac{20}{c}$, $c\neq 0$.

(a) The line $x=5$ is a vertical asymptote to the graph of f.

(i) Find the value of c.

(ii) Write down the equation of the horizontal asymptote to the graph of f. [4]

(b) The line $y=h$, where $h\in\mathbb{R}$, intersects the graph of $|f(x)|$ at exactly one point.

Find the possible values of h. [3]

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Section B (Skills/Concepts Practice)

[Maximum mark: 13]

Two points A and B have coordinates $(2, 3, 1)$ and $(5, 9, 3)$, respectively.

(a) (i) Find $\overrightarrow{AB}$.

(ii) Find $|\overrightarrow{AB}|$. [4]

Let $\overrightarrow{AC} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$.

(b) Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [4]

(c) Find the area of triangle ABC. [2]

(d) Hence or otherwise, find the shortest distance from C to the line through A and B. [3]

[Maximum mark: 15]

The first two terms of an infinite geometric sequence are $u_1 = 20$ and $u_2 = 16\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

(a) (i) Find an expression for r in terms of θ.

(ii) Find the possible values of r. [5]

(b) Show that the sum of the infinite sequence is $\dfrac{100}{3 + 2\cos 2\theta}$. [4]

(c) Find the values of θ which give the greatest value of the sum. [6]

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[Maximum mark: 17]

The mass M of oranges in grams is normally distributed with mean μ. The following table shows probabilities for values of M.

Values of M	$M < 110$	$110 \le M \le 220$	$M > 220$
$\mathrm{P}(X)$	0.02	0.96	k

(a) (i) Write down the value of k.

(ii) Show that $\mu = 165$. [4]

(b) Find $\mathrm{P}(M < 120)$. [5]

The oranges are packed in carton boxes of fifteen.

Any oranges with a mass less than 120 grams are classified as small.

(c) Find the probability that a box of oranges selected at random contains at most two small oranges. [3]

(d) A wooden crate contains 20 boxes of oranges. A crate is selected at random.

(i) Find the expected number of boxes in this crate that contain at most two small oranges.

(ii) Find the probability that at least 18 boxes in this crate contain at most two small oranges. [5]