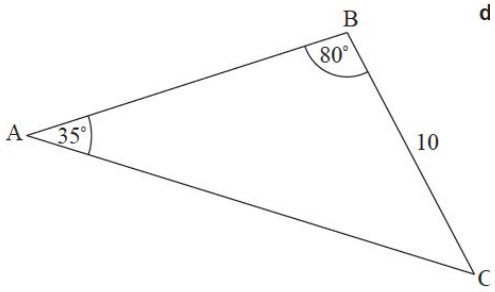
Math SL PROBLEM SET 102

Section A (Short Answer)

1. (CA) The following diagram shows triangle ABC.



$BC = 10\,\text{cm}$, $A\hat{B}C = 80°$ and $B\hat{A}C = 35°$.

   a. Find AC.
   b. Find the area of triangle ABC.

2. (CA) Let $u = 6\boldsymbol{i} + 3\boldsymbol{j} + 6\boldsymbol{k}$ and $v = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$.
   a. Find: (i) $u \cdot v$; (ii) $|u|$; (iii) $|v|$.
   b. Find the angle between u and v.

3. (CA) The following table shows the sales, y millions of dollars, of a company, x years after it opened. The relationship between the variables is modelled by the regression line with an equation in the form of $y = ax + b$.

Time after opening (x years)	2	4	6	8	10
Sales (y millions of dollars)	12	20	30	36	52

   a. (i) Find the value of a and of b.
      (ii) Write down the value of r.
   b. Hence estimate the sales in millions of dollars after seven years.

4. (CA) The third term in the expansion of $(x + k)^8$ is $63x^6$. Find the possible values of k.

5. (CA) Let $f(x) = e^{x+1} + 2$, for $-4 \leq x \leq 1$.

   a. Sketch the graph of f.

   b. The graph of f is translated by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to obtain the graph of a function g. Find an expression for $g(x)$.

6. (CA) Ramiro walks to work each morning. During the first minute he walks 80 metres. In each subsequent minute he walks 90% of the distance walked during the previous minute. The distance between his house and work is 660 metres. Ramiro leaves his house at 08:00 and has to be at work by 08:15. Explain why he will not be at work on time.

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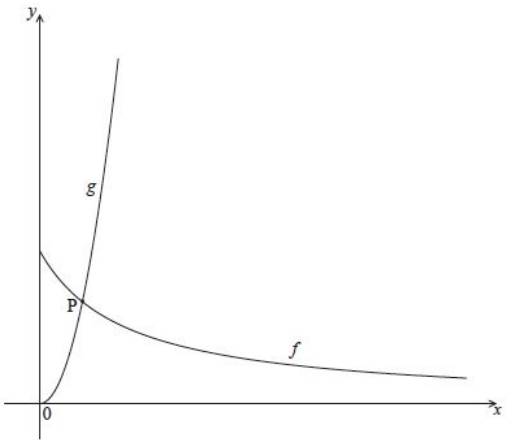
7. (CA) Let $f(x) = kx^2 + kx$ and $g(x) = x - 0.8$. The graphs of f and g intersect at two distinct points. Find the possible values of k .

Section B (Extended Response)

8. (CA) Let $f(x) = \frac{9}{x+2}$ and $g(x) = 3x^2$, for $x \geq 0$. Parts of the graphs of f and g are shown in the following diagram. The graphs of f and g intersect at the point $P(p, q)$.



   a. Find the value of p and of q .
   b. Write down $f'(p)$.

   Let L be the normal to the graph of f at P.

   c. (i) Find the equation of L, giving your answer in the form $y = ax + b$.
      (ii) Write down the y-intercept of L.

   d. Let ***R*** be the region enclosed by the y-axis, the graph of g and the line L. Find the area of ***R***.

9. A machine manufactures a large number of nails. The length, L mm, of a nail is normally distributed, where $L \sim N(50, \sigma^2)$.

   a. Find $P(50 - \sigma < L < 50 + 2\sigma)$.
   b. The probability that the length of a nail is less than 53.92 mm is 0.975. Show that $\sigma = 2.00$ (correct to three significant figures).

   All nails with length at least t mm are classified as large nails.

   c. A nail is chosen at random. The probability that it is a large nail is 0.75. Find the value of t .

   d. (i) A nail is chosen at random from the large nails. Find the probability that the length of this nail is less than 50.1 mm.
      (ii) Ten nails are chosen at random from the large nails. Find the probability that at least two nails have a length that is less than 50.1 mm.

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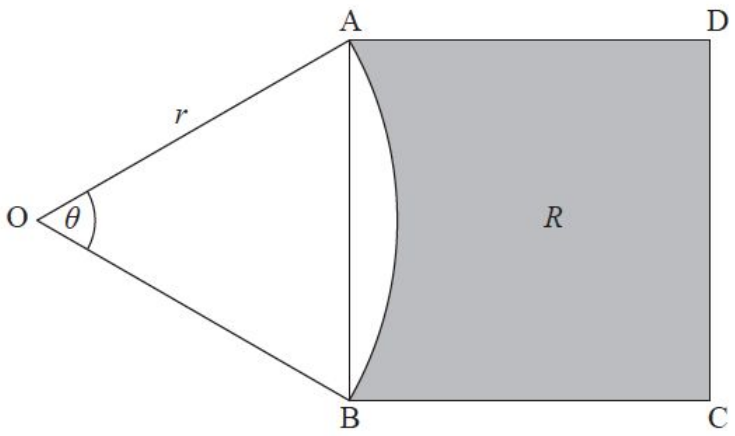
10. The following diagram shows a square ABCD, and a sector OAB of a circle centre O, radius r .
Part of the square is shaded and labelled R .



$A\hat{O}B = \theta$, where $0.5 \leq \theta < \pi$.

a. Show that the area of the square ABCD is $2r^2(1-\cos\theta)$.

b. When $\theta = \alpha$, the area of the square ABCD is equal to the area of the sector OAB .
(i) Write down the area of the sector when $\theta = \alpha$.
(ii) Hence find α .

c. When $\theta = \beta$, the area of R is more than twice the area of the sector. Find all possible values of β .