1. [Maximum mark: 6]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

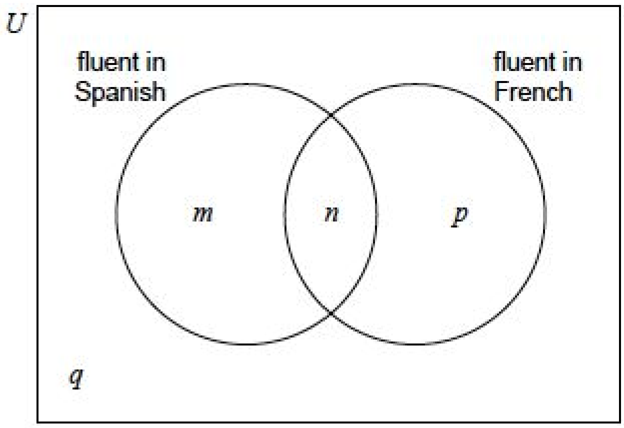
(b) Find the first term. [2]

(c) Find the sum of the first 20 terms. [2]

2. [Maximum mark: 6]

In a class of 30 students, 18 are fluent in Spanish, 10 are fluent in French, and 5 are not fluent in either of these languages. The following Venn diagram shows the events "fluent in Spanish" and "fluent in French".

The values m, n, p and q represent numbers of students.



(a) Write down the value of q. [1]

(b) Find the value of n. [2]

(c) Write down the value of m and of p. [3]

3. [Maximum mark: 7]

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g.

(a) Find the value of b. [3]

(b) The graph of $f(x) = x^2$ is transformed to obtain the graph of g.

Describe this transformation. [4]

4. [Maximum mark: 6]

Consider $\binom{11}{a} = \frac{11!}{a!\,9!}$.

(a) Find the value of a. [2]

(b) Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x+3)^{11}$. [4]

5. [Maximum mark: 6]

Consider the function f, with derivative $f'(x) = 2x^2 + 5kx + 3k^2 + 2$ where $x,\ k \in \mathbb{R}$.

(a) Show that the discriminant of $f'(x)$ is $k^2 - 16$. [2]

(b) Given that f is an increasing function, find all possible values of k. [4]

6. [Maximum mark: 8]

Let $f(x) = 4\cos\left(\frac{x}{2}\right) + 1$, for $0 \le x \le 6\pi$. Find the values of x for which $f(x) > 2\sqrt{2} + 1$.

7. [Maximum mark: 6]

Let X and Y be normally distributed with $X \sim \mathrm{N}(14, a^2)$ and $Y \sim \mathrm{N}(22, a^2)$, $a > 0$.

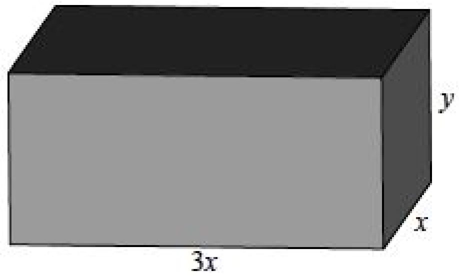
(a) Find b so that $\mathrm{P}(X > b) = \mathrm{P}(Y < b)$. [2]

It is given that $\mathrm{P}(X > 20) = 0.112$.

(b) Find $\mathrm{P}(16 < Y < 28)$. [4]

8. [Maximum mark: 14]

A small cuboid box has a rectangular base of length $3x$ cm and width x cm, where $x > 0$.
The height is y cm, where $y > 0$.



The sum of the length, width and height is $12\,\text{cm}$.

(a) Write down an expression for y in terms of x. [1]

The volume of the box is $V\,\text{cm}^3$.

(b) Find an expression for V in terms of x. [2]

(c) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

(d) (i) Find the value of x for which V is a maximum.

(ii) Justify your answer. [7]

(e) Find the maximum volume. [2]

9. [Maximum mark: 17]

The points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Let O be the origin.

(a) Find, in terms of k,

(i) $\overrightarrow{OA} \cdot \overrightarrow{OC}$;

(ii) $\overrightarrow{OB} \cdot \overrightarrow{OC}$. [3]

(b) Given that $\mathrm{A\hat{O}C} = \mathrm{B\hat{O}C}$, show that $k = 7$. [8]

(c) Calculate the area of triangle AOC. [6]

10. [Maximum mark: 14]

Let $g(x) = p^x + q$, for $x, p, q \in \mathbb{R}, p > 1$. The point $A(0, a)$ lies on the graph of g.

Let $f(x) = g^{-1}(x)$. The point B lies on the graph of f and is the reflection of point A in the line $y = x$.

(a) Write down the coordinates of B. [2]

The line L_1 is tangent to the graph of f at B.

(b) Given that $f'(a) = \frac{1}{\ln p}$, find the equation of L_1 **in terms of** x, p and q. [5]

The line L_2 is tangent to the graph of g at A and has equation $y = (\ln p)x + q + 1$.

The line L_2 passes through the point $(-2, -2)$.

The gradient of the normal to g at A is $\frac{1}{\ln\left(\frac{1}{3}\right)}$.

(c) Find the equation of L_1 in terms of x. [7]