

Math SL EXPLORATION LAB 10

Working with Composite Functions.

In this assignment, you will be introduced to taking integrals using the method of substitution and will see its connection to the method of differentiating using the Chain Rule.

1. SKILL REVIEW: Use the Chain Rule to differentiate the following composite functions:

- a. (i) $y = (2x^2 + 6x + 5)^5$ (ii) $y = \sqrt[3]{2x^3 + x + 1}$ (iii) $y = \frac{1}{(x^3 + x - 1)^2}$
- b. (i) $y = \sin(x^2) + \sin^2(x)$ (ii) $y = \tan(2x) + \frac{1}{\sin(x)}$ (iii) $y = \sin(\sqrt{x})$
- c. (i) $y = e^{2x+1}$ (ii) $y = 2e^{4-3x^2}$ (iii) $y = \sqrt{e^x}$
- d. (i) $y = \ln(x^2 + 1)$ (ii) $y = \ln(\sin(x) + x)$ (iii) $y = \ln(e^x - e^{-x})$

2. SKILL REVIEW: Deriving and Integrating with Composite Functions.

- a. Show that $\frac{d}{dx}(2x+3)^4 = 8(2x+3)^3$ and hence $\int(2x+3)^3 dx$.
- b. Show that $\frac{d}{dx}\ln(5-2x) = \frac{-2}{5-2x}$ and hence $\int\frac{1}{5-2x} dx$.
- c. Show that $\frac{d}{dx}\tan(3x+\pi) = \frac{1}{2\cos^2(3x+\pi)}$ and hence $\int\frac{1}{\cos^2(3x+\pi)} dx$.
- d. Show that $\frac{d}{dx}(2x^3+3)^4 = 24x^2(2x^3+3)^3$ and hence $\int x^2(2x^3+3)^3 dx$.
- e. Show that $\frac{d}{dx}\ln(5-2x^2) = \frac{-4x}{5-2x^2}$ and hence $\int\frac{x}{5-2x^2} dx$.
- f. Show that $\frac{d}{dx}\cos(3x^4+\pi) = -12x^3\sin(3x^4+\pi)$ and hence $\int x^3\sin(3x^4+\pi) dx$.

3. NEW SKILL: Examples: Integration by Substitution:

Math SL EXPLORATION LAB 10

a. EXAMPLES:

$$1) \int -15x^4(-3x^5 - 1)^5 dx; u = -3x^5 - 1$$

$$2) \int -16x^3(-4x^4 - 1)^{-5} dx; u = -4x^4 - 1$$

$$3) \int -\frac{8x^3}{(-2x^4 + 5)^5} dx; u = -2x^4 + 5$$

$$4) \int (5x^4 + 5)^{\frac{2}{3}} \cdot 20x^3 dx; u = 5x^4 + 5$$

b. PRACTICE:

$$5) \int \frac{(5 + \ln x)^5}{x} dx; u = 5 + \ln x$$

$$6) \int 4 \sec 4x \cdot \tan 4x \cdot \sec^4 4x dx; u = \sec 4x$$

$$7) \int 36x^3(3x^4 + 3)^5 dx; u = 3x^4 + 3$$

$$8) \int x(4x - 1)^4 dx; u = 4x - 1$$

c. EXAMPLES:

$$9) \int -9x^2(-3x^3 + 1)^3 dx$$

$$10) \int 12x^3(3x^4 + 4)^4 dx$$

$$11) \int -12x^2(-4x^3 + 2)^{-3} dx$$

$$12) \int (3x^5 - 3)^{\frac{3}{5}} \cdot 15x^4 dx$$

d. PRACTICE:

$$13) \int (-2x^4 - 4)^4 \cdot -32x^3 dx$$

$$14) \int (e^{4x} - 4)^{\frac{1}{5}} \cdot 8e^{4x} dx$$

$$15) \int x(4x + 5)^3 dx$$

$$16) \int 5x\sqrt{2x + 3} dx$$

Math SL EXPLORATION LAB 10

4. NEW SKILL: Practice: Integration by Substitution:

http://www.teaching.martahidegkuti.com/shared/lnotes/6_calculus/integral/substitution/substitutionb.pdf

Oxford Exercise 9F, p302, Q1 - 12