IM2 Problem Set 6.9 - Working with Quadratic Functions

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic functions? • How can I use graphs and equations of quadratic relations to make predictions from data sets & their models

1. (CI) For the following parabolas:

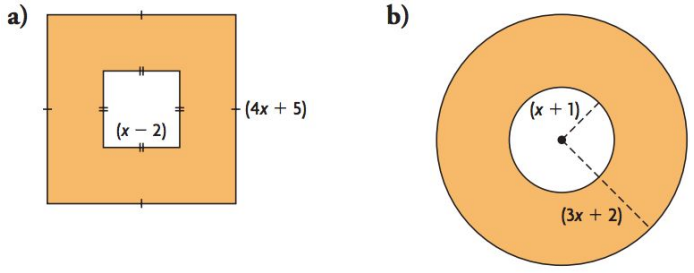
   (i) $f(x) = 2x^2 - 20x + 41$ (ii) $g(x) = -4x^2 - 12x + 7$

   a. find the axis of symmetry
   b. find the coordinates of the vertex.
   c. rewrite the equation in vertex form.
   d. find the x- and y-intercepts
   e. sketch the parabola

2. (CI) For the following quadratic functions, expand/simplify so that the final equation of the quadratic is presented in standard form.
   a. (i) $y = 2(x + 1)(2x + 3)$ (ii) $y = ½ (3x - 2)(4x - 5)$
   b. (i) $y = -3(x + 3)^2 + 6$ (ii) $y = ¼ (2x + 5)^2 - 10$

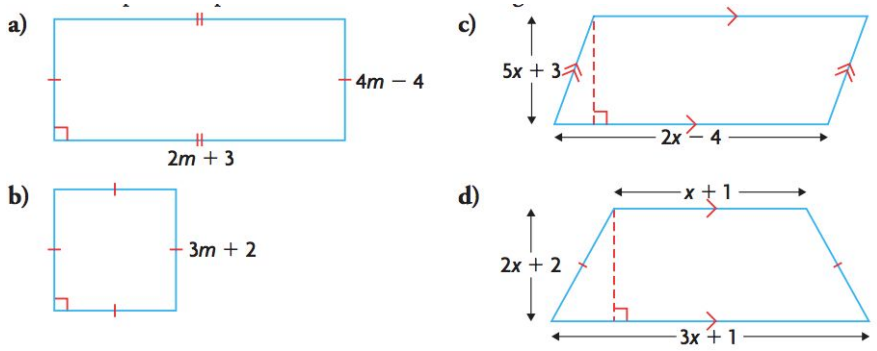
3. (CI) Factor the following quadratic equations and then find the x-intercepts of the parabola.
   a. (i) $y = x^2 + 6x - 40$ (ii) $y = 3x^2 + 24x + 45$ (iii) $y = 9x^2 - 25$
   b. (i) $y = 4x^2 - 16x + 15$ (ii) $y = 2x^2 + 5x - 12$ (iii) $y = 6x^2 + 5x + 1$

4. (CI) Determine a simplified expression for the shaded area in each figure.



5. (CA) Determine the equations of the following parabolas:
   a. The vertex is at (-3,2) and the parabola passes through (5,9)
   b. The zeroes are at $x = 5$ and $x = -3$ and the parabola passes through (0,-45)
   c. The zeroes are at $x = 2.5$ and $x = -1.5$ and the parabola passes through (-1,14)

6. (CI) Write a simplified expression for the area of each figure:



7. (CA) Accountants for the HiTech Shoe Company have determined that the quadratic relation $P(x) = -2x^2 + 24x - 54$ models the company's profits for the next year. In this relation, P represents the profit (in \$100,000s) and x represents the number of shoes sold (in 100,000s)

    a. If P(4) = 10, explain what the point (4,10) means in this question.
    b. Find the y-intercept and give one reason that the y-intercept is negative.
    c. Express the equation in factored form.
    d. State the window settings you used to graph the relation.
    e. Where are the zeroes and what do they represent?
    f. Determine the number of shoes that must be sold to maximize the profits? What are the maximum profits of the company?

8. (CI) For the following quadratic relations, determine (i) the axis of symmetry, (ii) the vertex and (iii) the y-intercept and (iv) the x-intercept(s) and then sketch the parabola:

    a. $y = -2x^2 + 12x - 10$ b. $y = 6x^2 - 15x + 6$

EXTENSION PROBLEMS

9. (CA) The underside of a concrete railway underpass forms a parabolic arch. The arch is 30.0 m wide at its base and 10.8 m high in the center. The upper surface of the underpass is 40.0 m wide and concrete is 2.0 m thick at the center. Can a truck that is 5.0 m wide and 7.5 m tall get through this underpass?

10. (CA) The Next Cup Coffee Shop sells its coffee for \$2.60 per mug and at that price, the shop sells 200 cups of coffee. However, it is known (through marketing research) that every \$0.05 decrease in the price of a cup of coffee, the shop will sell 10 more cups of coffee. Determine the price for a cup of coffee that will maximize the daily revenue from the sales of coffee. What is the maximum revenue that the shop will have at this price?