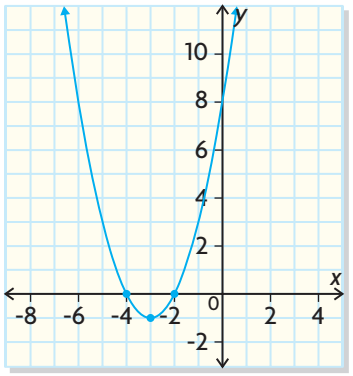
3.4 Expanding Quadratic Expressions

GOAL

Determine the product of two binomials using a variety of strategies.

YOU WILL NEED

- algebra tiles



LEARN ABOUT the Math

Brandon was doing his math homework. For one question, he had to determine the equation of the parabola shown at the right.

Brandon's answer was $y = (x + 4)(x + 2)$.

His older sister, Devin, said that the answer can also be $y = x^2 + 6x + 8$.

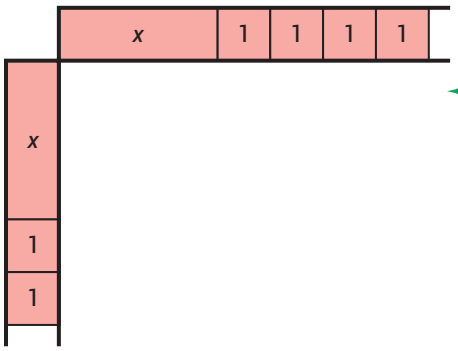
? How can Devin show Brandon that both answers are correct?

EXAMPLE 1 Connecting an area model to the product of two binomials

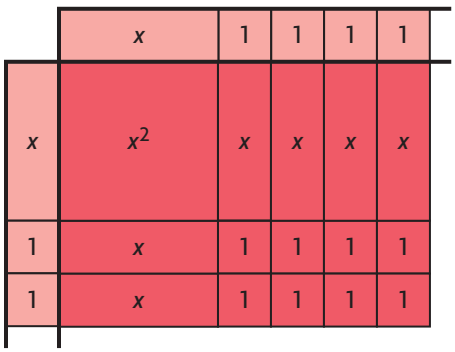
Show that the equations $y = (x + 4)(x + 2)$ and $y = x^2 + 6x + 8$ represent the same quadratic relation.

Devin's Solution

$y = (x + 4)(x + 2)$



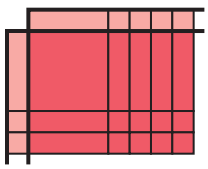
I wanted to show Brandon how to multiply two binomials. I know that the area of a rectangle is the product of its length and its width. I used algebra tiles to represent a width of $x + 2$ and a length of $x + 4$.



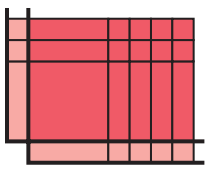
I used x^2 tiles, x tiles, and unit tiles to fill in the area of the rectangle with these dimensions. The area of the rectangle represents the product of the two binomials.

Communication Tip

The tiles used for each dimension of a rectangle can be placed on either the left or right, and either on the top or bottom. The resulting area of the rectangle is the same in each case. The only difference occurs in the position of the x^2, x, and unit tiles within the rectangle. For example,



and



represent the same product.

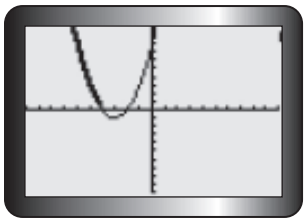
x^2	x	x	x	x
x	1	1	1	1
x	1	1	1	1

I counted the tiles in the rectangle to get an expression for its area, A.

$A = x^2 + 2x + 4x + 8$
$A = x^2 + 6x + 8$

$y = (x + 4)(x + 2)$ and $y = x^2 + 6x + 8$ are the same quadratic relation.

Brandon's equation is in factored form and mine is in standard form.



I graphed both relations to see if they represented the same parabola. The second parabola traced exactly over the first parabola.

Reflecting

	x	4
x	x^2	$4x$
2	$2x$	8

A. Why did Devin use only red tiles in her rectangle model?

B. Explain how the area diagram at the left is related to Devin's algebra tile model and the product $(x + 4)(x + 2)$.

C. Is the value of a always the same in factored form and standard form if both relations represent the same parabola? Explain.

APPLY the Math

EXAMPLE 2 Connecting the product of two binomials to the distributive property

Expand and simplify.

a) $(2x + 3)(x - 2)$ **b)** $(2x - 1)(x - 3)$

Lorna's Solution

a)

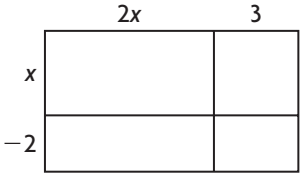
	x	x	1	1	1
x	x^2	x^2	x	x	x
-1	$-x$	$-x$	-1	-1	-1
-1	$-x$	$-x$	-1	-1	-1

I placed tiles that correspond to the binomial factors along the sides of a rectangle. I represented $x - 2$ as $x + (-2)$ because I didn't know how to remove part of a tile.

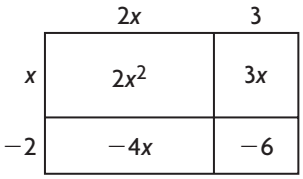
Then I used tiles to fill in the area. The rules for multiplying integers helped me choose the correct colours to use. Since a blue tile is negative and a red tile is positive, I used blue tiles to represent the negative product.

$$(2x + 3)(x - 2) = 2x^2 - 4x + 3x - 6$$
$$= 2x^2 - x - 6$$

I counted the different types of algebra tiles to get the product.



I noticed that the area in the tile model was divided into four sections, so I divided a rectangle into four small rectangles. I labelled the side lengths.



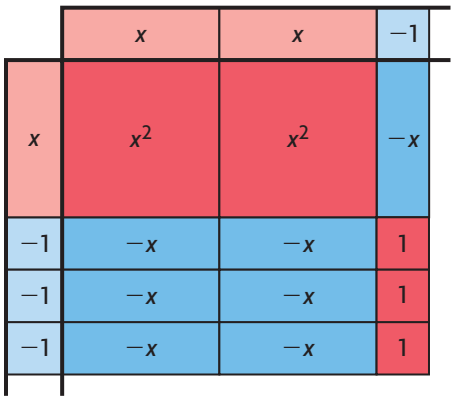
I wrote an expression for the area of each small rectangle. The area of the large rectangle is the sum of the areas of the four small rectangles. When I collected like terms, I saw that the product was the same.

$$(2x + 3)(x - 2) = 2x^2 - 4x + 3x - 6$$
$$= 2x^2 - x - 6$$

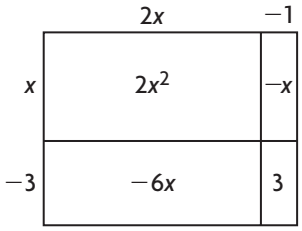
$$(2x + 3)(x - 2) = 2x(x - 2) + 3(x - 2)$$
$$= 2x^2 - 4x + 3x - 6$$
$$= 2x^2 - x - 6$$

I recognized the distributive property in the area model. The areas in the first column show the product $2x(x - 2)$. The areas in the second column show the product $3(x - 2)$. I used the distributive property again. Then I collected like terms to get the final result.

b)



I created an algebra tile model. This time the unit tiles that I used to fill in the area had to be positive red tiles, since the result is the product of two negative blue tiles.



I made an area diagram to show the area of the four sections of the tile model.

$$(2x - 1)(x - 3) = 2x(x - 3) - 1(x - 3)$$
$$= 2x^2 - 6x - x + 3$$
$$= 2x^2 - 7x + 3$$

I could have used the distributive property without a picture or model. I collected like terms to get the final result.

EXAMPLE 3 **Representing** the product of two binomials symbolically

Multiply each expression.

a) $(x - 5)(x + 5)$ **b)** $(3x - 5)^2$

Zac's Solution

a) $(x - 5)(x + 5) = x^2 + 5x - 5x - 25$
$= x^2 - 25$

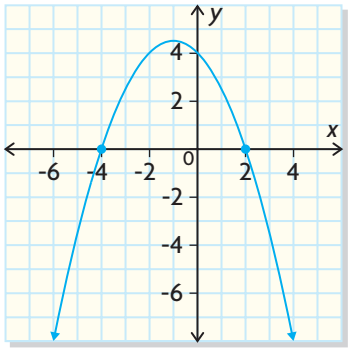
I multiplied each term in the second binomial by x and then by -5. I collected like terms and got a binomial for my final result.

b) $(3x - 5)^2 = (3x - 5)(3x - 5)$
$= 9x^2 - 15x - 15x + 25$
$= 9x^2 - 30x + 25$

I wrote the expression as a product of two binomials. I multiplied each term in the second binomial by $3x$ and then by -5. I collected like terms and got a trinomial for my final result.

EXAMPLE 4 **Connecting** the factored form and standard form of a quadratic relation

Determine the equation of the parabola. Express your answer in standard form.



Mathieu's Solution

$y = a[x - (-4)](x - 2)$
$y = a(x + 4)(x - 2)$

I wrote the equation in factored form using the zeros of the parabola. Then I wrote an equivalent expression for $x - (-4)$.

$x = 0, y = 4$
$4 = a(0 + 4)(0 - 2)$
$4 = a(4)(-2)$
$4 = -8a$
$\frac{4}{-8} = \frac{-8a}{-8}$
$-0.5 = a$

There is only one value of a that gives a parabola with these zeros and y-intercept. To determine this value, I substituted the coordinates of the y-intercept (0, 4) into the equation and solved for a.

$y = -0.5(x + 4)(x - 2)$
$y = -0.5(x^2 - 2x + 4x - 8)$
$y = -0.5x^2 + x - 2x + 4$
$y = -0.5x^2 - x + 4$

I substituted the value of *a* into the factored form of the equation. I multiplied the two binomials. Then I multiplied all the terms by −0.5 and collected like terms to get the result in standard form.

In Summary

Key Ideas

- Quadratic expressions can be expanded using the distributive property, then simplified by collecting like terms.
- An area diagram or algebra tiles can be used to show the relation between two binomial factors of degree one and their product.

Need to Know

- To calculate the product of two binomials, use the distributive property twice.

	ax	b
cx	acx^2	bcx
d	adx	bd

$$(ax + b)(cx + d) = ax(cx + d) + b(cx + d)$$
$$= acx^2 + adx + bcx + bd$$

CHECK Your Understanding

1. State the binomials that are represented by the length and width of each rectangle. Then determine the product that is represented by the area.

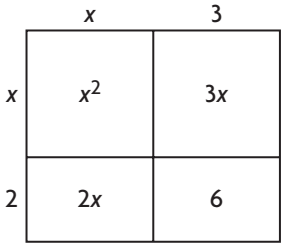
a)

	x	1	1	1	1	1
x	x^2	x	x	x	x	x
1	x	1	1	1	1	1

b)

	x	-1	-1
x	x^2	$-x$	$-x$
-1	$-x$	1	1
-1	$-x$	1	1

2. Copy and complete this table.

	Expression	Area Diagram	Expanded and Simplified Form
	$(x + 2)(x + 3)$		$x^2 + 5x + 6$
a)	$(x + 1)(x + 6)$		
b)	$(x + 1)(x - 4)$		
c)	$(x - 2)(x + 2)$		
d)	$(x - 3)(x - 4)$		
e)	$(x + 2)(x + 4)$		
f)	$(x - 2)(x - 6)$		

PRACTISING

3. Determine the missing terms.

a) $(m + 3)(m + 2) = \blacksquare + 2m + 3m + \bullet$

b) $(k - 2)(k + 1) = \blacksquare + \bullet - 2k - 2$

c) $(r + 4)(r - 3) = r^2 - 3r + \blacksquare - \bullet$

d) $(x - 5)(x - 2) = x^2 - \blacksquare - \bullet + 10$

e) $(2n + 1)(3n - 2) = \blacksquare - \bullet + 3n - 2$

f) $(5m - 2)(m - 3) = 5m^2 - \blacksquare - 2m + \bullet$

4. Expand and simplify.

a) $(x + 2)(x + 5)$ **c)** $(x + 2)(x - 3)$ **e)** $(x - 4)(x - 2)$

b) $(x + 2)(x + 1)$ **d)** $(x + 2)(x - 1)$ **f)** $(x - 5)(x - 3)$

5. Expand and simplify.

a) $(5x + 2)(x + 2)$ **c)** $(x - 2)(7x + 3)$ **e)** $(x - 2)(4x - 6)$

b) $(x + 2)(4x + 1)$ **d)** $(3x - 2)(x + 1)$ **f)** $(7x - 5)(x - 3)$

6. Expand and simplify.

a) $(x + 3)(x - 3)$ **c)** $(2x - 1)(2x + 1)$ **e)** $(4x - 6)(4x + 6)$

b) $(x + 6)(x - 6)$ **d)** $(3x - 3)(3x + 3)$ **f)** $(7x - 5)(7x + 5)$

7. Expand and simplify.

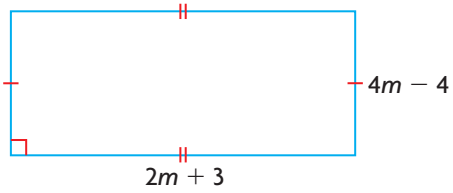
a) $(x + 1)^2$ **c)** $(c - 1)^2$ **e)** $(6z - 5)^2$

b) $(a + 4)^2$ **d)** $(5y - 2)^2$ **f)** $(-3d + 5)^2$

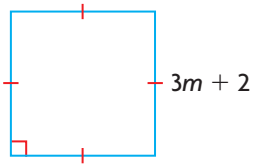
8. Write a simplified expression for the area of each figure.

A

a)



b)



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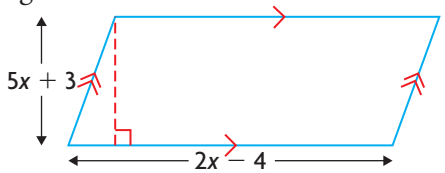
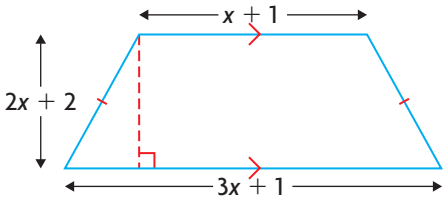


d)



9. Expand and simplify.

a) $4(x - 6)(x + 7)$

b) $-(x + 3)(4x - 1)$

c) $6x(x + 1)^2$

d) $(x + 4)(x - 2) + (x - 1)(x + 5)$

e) $(4x - 1)(4x + 1) - (x + 3)^2$

f) $2(3x + 4)^2 - 3(x - 2)^2$

10. Expand and simplify.

a) $(x + y)(2x + 3y)$

b) $(x + 2y)(3x + y)$

c) $(3x - 2y)(5x + 4y)$

d) $(8x - y)(7x + 2y)$

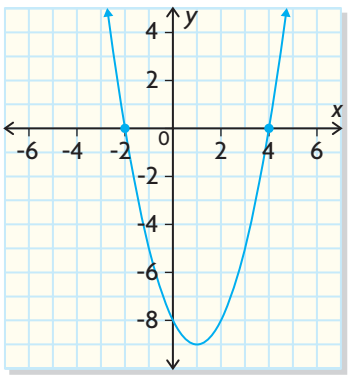
e) $(6x - 5y)(6x + 5y)$

f) $(9x - 7y)^2$

11. Determine the equation of each parabola. Express the equation in standard form.

K

a)



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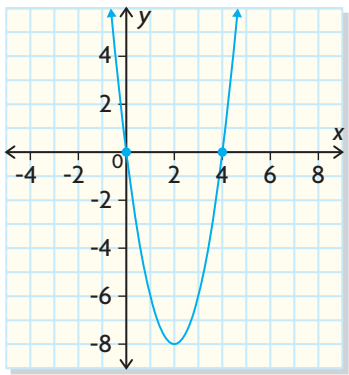
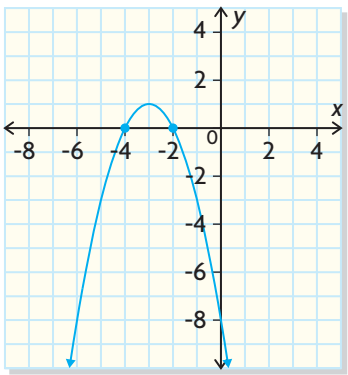
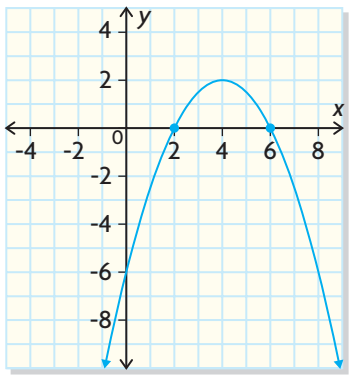


b)



d)



12. Write each quadratic relation in standard form. State which way the parabola opens.

	Zeros	A Point on the Graph
a)	-1 and 7	$(3, 5)$
b)	-1 and -5	$(-3, -4)$
c)	3 and 7	$(0, 3)$
d)	-2 and 6	$(-1, -1)$
e)	-2 and 8	$(3, 7)$

13. The area of a rectangle is represented by the expression $2x^2 + 14x + 20$. Bill claims that this rectangle could have either the dimensions $(2x + 4)$ and $(x + 5)$ or the dimensions $(2x + 10)$ and $(x + 2)$. Do you agree or disagree? Justify your opinion.

14. C Explain how you know that the product will be quadratic when you expand $(12x - 7)(5x + 1)$.

15. T The Rainbow Bridge in Utah, shown at the left, is a natural arch that is approximately parabolic in shape. The arch is about 88 m high. It is 84 m across at its base. Determine a quadratic relation, in standard form, that models the shape of the arch.

16. Jay claims that whenever two binomials are multiplied together, the result is always a trinomial. Is his claim correct? Use examples to support your decision.

Extending

17. Expand and simplify each expression.

a) $(x + 3)^3$
b) $(2x - 2)^3$
c) $(4x + 2y)^3$
d) $[(x + 2)(x - 2)]^2$
e) $(x + 6)(x + 3)(x - 6)(x - 3)$
f) $(3x^2 + 6x - 1)^2$

18. Expand each expression.

a) $(a + b)^1$
b) $(a + b)^2$
c) $(a + b)^3$
d) $(a + b)^4$

19. Discuss any patterns you see in question 18.