5 Chapter Review

FREQUENTLY ASKED Questions

Q: What information do I need about the graph of a quadratic relation to write its equation in vertex form?

A: You can write the equation in vertex form if you know the coordinates of the vertex and one additional point on the graph.

Study *Aid*

- See Lesson 5.4, Examples 1 to 3.
- Try Chapter Review Questions 8 to 10.

Q: What kinds of problems can you solve using the vertex form of a quadratic relation?

A: The vertex form of a quadratic relation is usually the easiest form to use when you need to determine the equation of a parabola of good fit on a scatter plot. The vertex form is also useful when you need to determine the maximum or minimum value of a quadratic relation.

Study *Aid*

- See Lesson 5.5, Examples 1, 2, and 4.
- Try Chapter Review Questions 11 to 13.

Q: How can you relate the standard form of a quadratic relation to its vertex form?

A: The standard form of a quadratic relation, $y = ax^2 + bx + c$, can be rewritten in vertex form if you know the value of a and the coordinates of the vertex:

- If the quadratic relation can be written in factored form, you can determine the zeros by setting each factor equal to zero. Calculating the mean of the x-coordinates of the zeros gives you the axis of symmetry and the x-coordinate of the vertex. Substitute the x-coordinate of the vertex into the quadratic relation to determine the y-coordinate of the vertex.
- If $y = ax^2 + bx + c$ cannot be factored, you can use partial factoring to express the equation in the form $y = x(ax + b) + c$. Solving $x(ax + b) = 0$ gives two points with the same y-coordinate, c. Calculating the mean of the x-coordinates of these points gives you the axis of symmetry and the x-coordinate of the vertex. Substitute the x-coordinate of the vertex into the quadratic relation to determine the y-coordinate of the vertex.
- If you graph $y = ax^2 + bx + c$ using graphing technology, then you can approximate the vertex from the graph or determine it exactly, depending on the features of the technology.

In all cases, after you know the vertex, you can use the value of a from the standard form of the relation to write the relation in vertex form, $y = a(x - h)^2 + k$.

Study *Aid*

- See Lesson 5.5, Example 3, and Lesson 5.6, Examples 1 and 2.
- Try Chapter Review Questions 14 to 17.

PRACTICE Questions

Lesson 5.1

1. Write the equations of two different quadratic relations that match each description.

a) The graph has a narrower opening than the graph of $y = 2x^2$.

b) The graph has a wider opening than the graph of $y = -0.5x^2$.

c) The graph opens downward and has a narrower opening than the graph of $y = 5x^2$.

2. The point (p, q) lies on the parabola $y = ax^2$. If you did not know the value of a, how could you use the values of p and q to determine whether the parabola is wider or narrower than $y = x^2$?

Lesson 5.2

3. Match each translation with the correct quadratic relation.

a) 3 units left, 4 units down

b) 2 units right, 4 units down

c) 5 units left

d) 3 units right, 2 units up

i) $y = (x - 3)^2 + 2$ **iii)** $y = (x - 2)^2 - 4$

ii) $y = (x + 3)^2 - 4$ **iv)** $y = (x + 5)^2$

Lesson 5.3

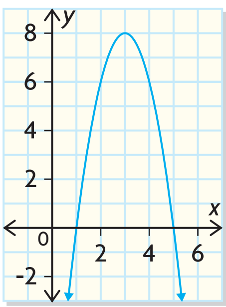
4. Which equation represents the graph shown? Explain your reasoning.

a) $y = -3(x + 3)^2 + 8$

b) $y = -3(x - 3)^2 + 8$

c) $y = 3(x - 3)^2 - 8$

d) $y = -2(x - 3)^2 + 8$



5. The parabola $y = x^2$ is transformed in two different ways to produce the parabolas $y = 2(x - 4)^2 + 5$ and $y = 2(x - 5)^2 + 4$. How are these transformations the same, and how are they different?

6. Blake rotated the parabola $y = x^2$ by 180° around a point. The new vertex is $(6, -8)$. What is the equation of the new parabola?

7. Reggie used transformations to graph $y = -2(x - 4)^2 + 3$. He started by reflecting the graph of $y = x^2$ in the x-axis. Then he translated the graph so that its vertex moved to (4, 3). Finally, he stretched the graph vertically by a factor of 2.

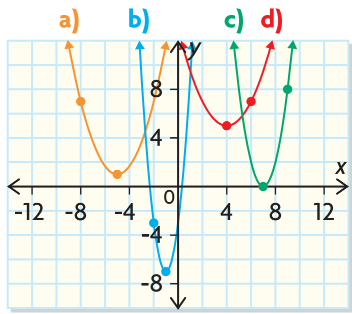
a) Why was Reggie's final graph not correct?

b) What sequence of transformations should he have used?

c) Use transformations to sketch $y = -2(x - 4)^2 + 3$ on grid paper.

Lesson 5.4

8. Use the point marked on each parabola, as well as the vertex of the parabola, to determine the equation of the parabola in vertex form.



9. Use the given information to determine the equation of each quadratic relation in vertex form.

a) vertex at $(-3, 2)$, passes through $(-1, 4)$

b) vertex at $(1, 5)$, passes through $(3, -3)$

10. This table shows residential energy use by Canadians from 2002 to 2006, where 1 petajoule equals 1 000 000 000 000 000 joules.

Year	Residential Energy Use (petajoules)
2002	1286.70
2003	1338.20
2004	1313.00
2005	1296.60
2006	1250.30

a) Use technology to create a scatter plot and a quadratic regression model.
b) Determine the vertex, and write the equation of the model in vertex form.
c) According to your model, when was energy use at a maximum during this period?

Lesson 5.5

11. Karla hits a golf ball from an elevated tee to the green below. This table shows the height of the ball above the ground as it moves through the air.

Time (s)	Height (m)
0.0	30.00
0.5	41.25
1.0	50.00
1.5	56.25
2.0	60.00
2.5	61.25
3.0	60.00
3.5	56.25
4.0	50.00

a) Create a scatter plot, and draw a curve of good fit.
b) Estimate the coordinates of the vertex.
c) Determine a quadratic relation in vertex form to model the data.
d) Use the quadratic regression feature of graphing technology to create a model for the data. Compare this model with the model you created by hand for part c). How accurate is the model you created by hand?

12. A farming community collected data on the effect of different amounts of fertilizer, x, in 100 kg/ha, on the yield of carrots, y, in tonnes. The resulting quadratic regression model is $y = -0.5x^2 + 1.4x + 0.1$. Determine the amount of fertilizer needed to produce the maximum yield.

13. A local club alternates between booking live bands and booking DJs. By tracking receipts over a period of time, the owner of the club determined that her profit from a live band depended on the ticket price. Her profit, P, can be modelled using $P = -15x^2 + 600x + 50$, where x represents the ticket price in dollars.
a) Sketch the graph of the relation to help the owner understand this profit model.
b) Determine the maximum profit and the ticket price she should charge to achieve the maximum profit.

14. For each quadratic relation,
i) write the equation in factored form
ii) determine the coordinates of the vertex
iii) write the equation in vertex form
iv) sketch the graph

a) $y = x^2 - 8x + 15$
b) $y = 2x^2 - 8x - 64$
c) $y = -4x^2 - 12x + 7$

Lesson 5.6

15. Express each quadratic relation in vertex form using partial factoring to determine two points that are the same distance from the axis of symmetry.
a) $y = x^2 + 2x + 5$
b) $y = -x^2 + 6x - 3$
c) $y = -3x^2 + 42x - 147$
d) $y = 2x^2 - 20x + 41$

16. Write each quadratic relation in vertex form using an appropriate strategy.
a) $y = x^2 - 6x - 8$
b) $y = -2(x + 3)(x - 7)$
c) $y = x(3x + 12) + 2$
d) $y = -2x^2 + 12x - 11$

17. The height, h, of a football in metres t seconds since it was kicked can be modelled by $h = -4.9t^2 + 22.54t + 1.1$.
a) What was the height of the football when the punter kicked it?
b) Determine the maximum height of the football, correct to one decimal place, and the time when it reached this maximum height.