A. Lesson Context

BIG PICTURE of this UNIT:	• How do we measure "change" in a function or function model? • How do we analytically analyze a function or function model – beyond a simple preCalculus & visual/graphic level? • ?		
CONTEXT of this LESSON:	Where we've been We understand how to differentiate and work with polynomial , sinusoidal, exponential & log functions	Where we are How do we differentiate and work with functions that arise from a product or quotient of two other functions?	Where we are heading Working with more complicated functions that are variations of polynomials, sinusoidal and exp/log

B. Lesson Objectives

1. Find out how to take the derivative of a product of functions.
2. Find out how to take the derivative of a quotient of functions.
3. Use these differentiation methods to apply calculus skills (tangents/normals & curve sketching) to simple problems in curve sketching

C. Skill Development – Derivatives of a Product of Functions

Now, let's use desmos.com & wolframalpha.com to develop an understanding of the derivative of a product of 2 functions.

Examples to use ➔

1. Use $y = x\sin(x)$ which is a "new" function, made by taking the product of two functions (namely $f(x) = x$ and $g(x) = \sin(x)$, such that $y = f(x) \times g(x) = x\sin(x)$

   (a) So let's start with a prediction ➔ what do we predict the derivative of $y = x\sin(x)$ to be? Use desmos to graph $y = x\sin(x)$ and its derivative as well as your proposed derivative. Make observation.

   (b) Now use wolframalpha and ask wolframalpha to give us the derivative of $y = x\sin(x)$ ➔ now how do we understand HOW that derivative came about?

2. So, now make a prediction for the derivative of $y = x^2e^x$. Use wolframalpha to confirm your prediction.

D. Skill Development – Derivatives of a Quotient of Functions

Now, let's use desmos.com & wolframalpha.com to develop an understanding of the derivative of a product of 2 functions.

Examples to use ➔

1. Use $y = \frac{e^x}{x+1}$ which is a "new" function, made by taking the quotient of two functions (namely $f(x) = e^x$ and $g(x) = x+1$, such that $y = \frac{f(x)}{g(x)} = \frac{e^x}{x+1}$

   (a) So let's start with a prediction ➔ what do we predict the derivative of $y = \frac{e^x}{x+1}$ to be? Use desmos to graph $y = \frac{e^x}{x+1}$ and its derivative as well as your proposed derivative. Make observation.

   (b) Now use wolframalpha and ask wolframalpha to give us the derivative of $y = \frac{e^x}{x+1}$ ➔ now how do we understand HOW that derivative came about?

2. So, now make a prediction for the derivative of $y = \frac{\cos(x)}{x^2+1}$. Use wolframalpha to confirm your prediction.

E. Skill Development – Differentiation Techniques: Summary

Rule	"formula"
Product Rule	
Quotient Rule	

F. Problem 2 – Applying Calculus: Working with Rates of Change & Tangents & Normals

1. Given the function $f(x) = (3x+1)\ln(x)$, determine:
   i. The equation of the derivative
   ii. The exact value of the instantaneous rate of change at $x = e$.

2. Find the equation of the derivative of the following functions:
   i. $h(x) = \dfrac{\sin(x)}{x^3 - 2}$
   ii. $f(x) = \dfrac{5x+3}{x^2+1}$
   iii. $b(x) = \sqrt{x}\left(4x^2 - 2x\right)$
   iv. $a(x) = \dfrac{3x^2 + 2x + 1}{x^3}$

3. (CI) Determine the equation of the line tangent to $y = x^3 \ln(x)$ at $x = 1$.

4. Find $\dfrac{dy}{dx}$ for $y = \dfrac{x^2 - 1}{2x^2 + 1}$. Determine the values of x for which $\dfrac{dy}{dx} > 0$.

5. (CI) Determine the equation of the tangent to $y = \left(2x^3 - 4x + 2\right)\left(x^2 - 3x + 1\right)$ at the point (2,-10).

6. (CI) Find the point(s) where the tangent line to the curve of $f(x) = \dfrac{x^2 - 2x + 4}{x^2 + 4}$ is horizontal.

7. Find the minimal value of $g(x) = \dfrac{e^x}{x};\ x > 0$.

8. (CI) For the curve defined by $g(x) = e^{-x}\cos(x)$ on the domain of $\left[0, \frac{5\pi}{2}\right]$, determine:

   i. the x- and y-intercept(s);
   ii. the first two stationary points;
   iii. the “nature” of these stationary points (max/min/neither);
   iv. hence, sketch the function $g(x) = e^{-x}\cos(x)$.

9. (**CI**) For the curve of $g(x) = e^{x}\sin(x)$,

   i. Find the equations of $g'(x)$ and $g''(x)$.
   ii. Find the values of *x* for which $g'(x) = 0$ and $g''(x) = 0$.
   iii. Given your work in Qi. and Qii., determine the intervals of increase and decrease and classify the extrema.
   iv. Sketch the function, given your work in Qiii.

10. (**CI**) Given the function $g(x) = xe^{x} - e^{x}$,
    i. Evaluate the exact values of: $(a)\ g(1) \qquad (b)\ g(0)$
    ii. Show that $\frac{d}{dx}g(x) = e^{x} + g(x)$.
    iii. Detemine the intervals of increase/decrease of $y = g(x)$.
    iv. Show that $y = g(x)$ has an inflection point at x = -1
    v. Determine the interval in which $y = g(x)$ is concave up.
    vi. Determine the equation of the tangent to the curve at *x* = 1.

11. Given the function $y = \frac{x}{3-2x}$;
    i. Determine the equation of the aymptotes of this rational function.
    ii. Hence or otherwise, evaluate $\lim\limits_{x \to -\infty} \frac{x}{3-2x}$
    iii. Determine the x- and y-intercepts.
    iv. Find the equation of the line that is normal to the curve $y = \frac{x}{3-2x}$ at the point where *x* = 1.