Three parabolas lie in the Cartesian plane, P_1, P_2, and P_3. The equations for the parabolas are as follows:

$P_1: f(x) = 2x^2 - 4x - 1$

$P_2: g(x) = 3x^2 - 18x + 30$, where the values of A, B, and C in Parabola 3 will be determined.

$P_3: h(x) = Ax^2 + Bx + C$

1. For Parabola 1, P_1, determine the location of the vertex.
2. **Hence, or otherwise**, write the equation for P_1 in vertex form ($f(x)=a(x-h)^2+k$).
3. State the interval of the domain in which f(x) is described as being an **increasing function**.
4. Algebraically, determine the roots (zeroes or x-intercepts) of y = f(x). Verify with technology.
5. **Hence**, write the equation for P_1 in factored form, in both approximate form (3 sds) and exact form.
6. Graph the parabola on the TI-84 & DESMOS. **Sketch** this parabola into your notes, labeling key points.

7. For Parabola 2, P_2, determine the location of the vertex.
8. **Hence, or otherwise**, write the equation for P_2 in vertex form ($g(x)=a(x-h)^2+k$).
9. This parabola has been transformed from the original parent function, $y = x^2$. Describe the transformations that were applied, given your work from Q8.
10. This parabola, P_2 will now be inverted.
    a. Explain what an "inverse" is.
    b. Will the inverted parabola represent a function? Why or why not?
    c. Determine an equation of the inverted parabola.
    d. Determine an equation of the inverted parabola such that it is a function.
    e. Compose the inverse function with the original function, $g \circ g^{-1}(x)$. What **should** happen? Why?
11. Is there a location or interval in which g(x) neither increases or decreases?
12. Graph the parabola on the TI-84 & DESMOS. **Sketch** this parabola into your notes, labeling key points.

You will now determine the equation of the third parabola, P_3, given the information provided.

13. Write the vertices of P_1 and P_2.
14. The three vertices of the parabolas all lie on the exponential function, $m(x) = K + L^x$, where K and L are constants that you need to determine. Given the location of the first two vertices of P_1 and P_2, determine the values of K and L and hence determine the vertex of P_3 (given that the x co-ordinates of the vertices are in an **arithmetic** progression.)
15. Write the values of the leading coefficients of P_1 and P_2 (call them a_1 and a_2). The value of the leading coefficient of P_3 can be determined as follows: The values of a_1, a_2 and a_3 represent three terms of a geometric sequence. Hence, determine the value of a_3.
16. Write the equation of P_3 in standard form.

You will now draw a line segment connecting the two vertices of the first two parabolas, P_1 and P_2. Call this line segment $\overline{V_1V_2}$.

17. Explain what the idea of being **tangent to a function** means. Is the line containing the segment $\overline{V_1V_2}$ **tangent to** the parabola P_2 at V_2? Are there lines that can be tangent to P_2 at this point (V_2)?
18. Determine the distance between the two vertices.
19. Hence or otherwise, determine the equation of the circle, wherein V_1 and V_2 are the endpoints of a diameter.
20. Determine the equation of the perpendicular bisector of $\overline{V_1V_2}$ (the line segment between the two vertices.)
21. Determine the angle of $\overline{V_1V_2}$.