- If $f'(x)$ changes from negative to positive at c, then point $(c, f(c))$ is a local minimum of f.
- If $f'(x)$ changes from positive to negative at c, then point $(c, f(c))$ is a local maximum of f.
- If $f'(x)$ does not change sign at c, then point $(c, f(c))$ is neither a maximum nor a minimum.

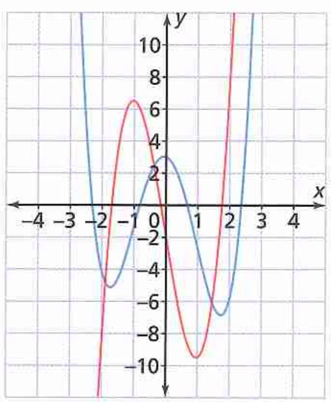
- **The First Derivative Test for Absolute Extrema**
  Let c be a critical number of a function f that is continuous over an interval D, the domain of f.
  - If $f'(x)$ is negative for all $x < c$ and $f((x)$ is positive for all $x > c$, then $f(c)$ is the absolute minimum of f.
  - If $f'(x)$ is positive for all $x < c$ and $f((x)$ is negative for all $x > c$, then $f(c)$ is the absolute maximum of f.

4.3 Exercises

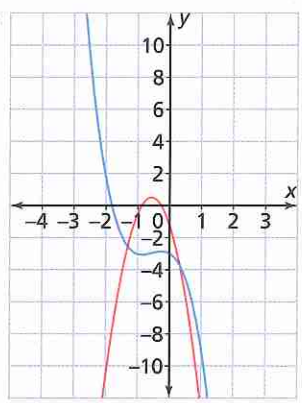
A

1. In each graph, which curve represents $y = f(x)$ and which represents $y = f'(x)$? Explain your choice.

(a)

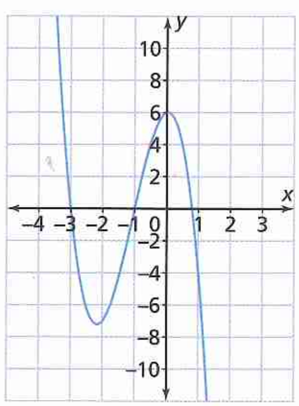


(b)

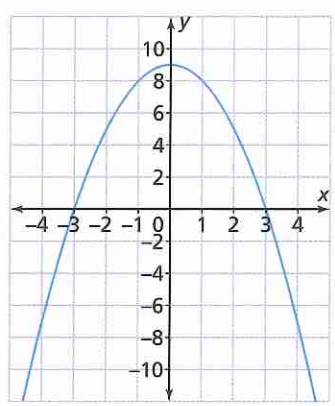


2. Each graph represents a function. Graph the derivative of each function.

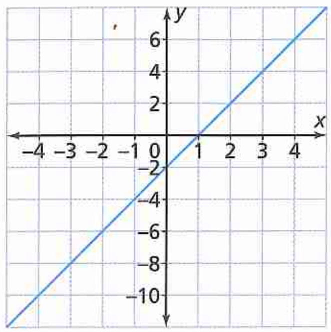
(a)

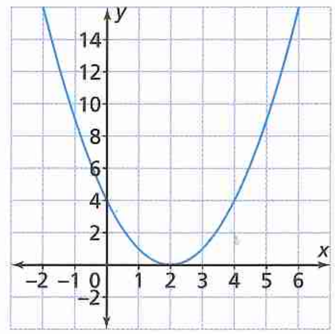


(b)



3. Each graph represents the derivative of a function. Graph a possible corresponding function.

(a) 

(b) 

4. As x increases over an interval, $f(x)$ increases and then decreases. Describe the behaviour of $f'(x)$. Sketch a possible graph of $f(x)$. What can you conclude about $f(x)$?

5. A polynomial function has three critical numbers: $x = -2$, $x = 1$, and $x = 4$. State the intervals on the domain created by these numbers.

B

6. For each function, find the critical numbers. Use the first derivative test to identify the local maximum and minimum values.
   (a) $g(x) = 2 - 6x - x^2$
   (b) $g(x) = 2x^3 - 9x^2 + 12x$
   (c) $g(x) = x^3 - 27x - 9$
   (d) $g(x) = x^4 - 2x^2 + 10$
   (e) $g(x) = 3x^4 - 4x^3 + 2$
   (f) $g(x) = 4x^4 - 4x^3 - 2x^2$
   (g) $g(x) = x^4 + 2x^3$
   (h) $g(x) = 12x^2 - 4x^3$

7. For each function, find the critical numbers. Determine where the function increases and decreases. Decide whether each critical point represents a maximum value, a minimum value, or neither. Use this information to graph the function.
   (a) $f(x) = x^2 - 4x + 5$
   (c) $f(x) = 10x - x^2$
   (c) $f(x) = x^3 - 3x^2 + 2$
   (d) $f(x) = x^3 - 3x + 6$
   (e) $f(x) = 2x^3 - 6x^2 - 18x + 3$
   (f) $f(x) = 2 - x^3$
   (g) $f(x) = x^4 + 4x$
   (h) $f(x) = x^4 - 6x^2 - 3$

8. **Knowledge and Understanding:** For $f(x) = x^4 - 32x + 4$, find the critical numbers, the intervals on which the function increases and decreases, and all the local extrema. Use graphing technology to verify your results.

9. Sketch a graph of the function g that is differentiable on the interval $-2 \le x \le 5$, decreases on $0 < x < 3$, and increases elsewhere on the domain. The absolute maximum of g on the first interval is 7 and the absolute minimum is -3. The graph of g has local extrema at $(0, 4)$ and $(3, -1)$.

10. **Communication:** Graph a quartic polynomial function that has four zeros, one absolute minimum, a different local minimum, and one local maximum.

11. Find a value of k that gives $f(x) = kx^2 - 4x + 6$ an absolute maximum at $x = -2$.

12. Find a value of k that gives $f(x) = x^2 + kx + 2$ a local minimum value of 1.

13. A publishing company uses the model $P(x) = 12x - 0.000\,1x^2 - 10\,000$ to estimate the profit, P, from the sale of x copies of a novel. The maximum print run for this novel is 10 000 books. How many books should be printed to maximize profit?

14. Four congruent squares are cut from the corners of a 5-cm by 8-cm piece of sheet metal. The metal is folded to form a small, open box. The volume, V, of the box is given by $V(x) = 4x^3 - 26x^2 + 40x$, where volume is measured in cubic centimetres and x is the length of each congruent square. What length x will produce a box with maximum volume?

15. **Application:** The table shows the number of students who are absent from a large high school on certain days with the flu. Using a polynomial model, determine when the absences were at a maximum and at a minimum during this two-week period.

Day	0	3	6	9	12	14
Students Away with Flu	96	204	239	172	55	32

16. **Thinking, Inquiry, Problem Solving:** During a rocket's flight, the velocity of the rocket is recorded at 1-s intervals. Use this data to model and graph the rocket's altitude versus time.

Time (s)	0	1	2	3	4	5	6	7	8	9	10
Velocity (m/s)	0	5	10	35	65	90	110	95	55	25	0

Time (s)	11	12	13	14	15	16	17	18	19	20
Velocity (m/s)	−10	−20	−30	−40	−50	−60	−70	−80	−90	0

17. **Check Your Understanding:** Does a function always have a local maximum or minimum at every critical number? Illustrate with one or more examples.

C 18. A rectangular pen will be built with fencing that costs \$25/m. The budget for the project is \$1500. What are the dimensions of the pen with the largest possible area?

19. Find the point on the graph of $f(x) = 4x^3 - 3x^2 + 2x - 3$ where the slope of the tangent line represents a minimum.