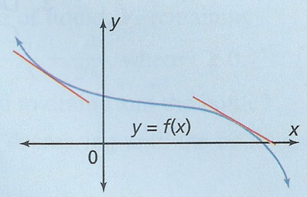
- A function $f(x)$ is **decreasing** on the open interval I $(a < x < b)$ if $f(x_1) > f(x_2)$ for all pairs of numbers, x_1 and x_2, such that $x_1 < x_2$ in I.
- For a function f that is continuous and differentiable on an interval I,
  - $f(x)$ is **increasing** if $f'(x) > 0$ for all x in I
  - $f(x)$ is **decreasing** if $f'(x) < 0$ for all x in I



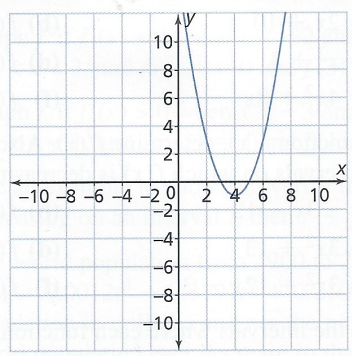
f(x) decreasing; f′(x) < 0

4.1 Exercises

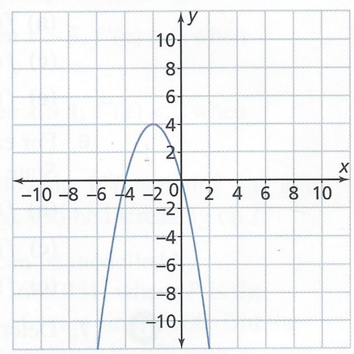
A

1. Identify the intervals on which the function increases or decreases.

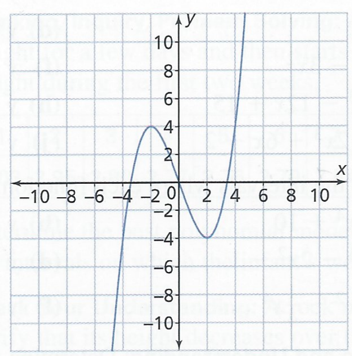
(a)



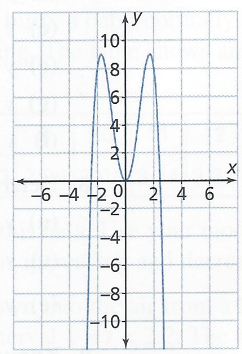
(b)



(c)



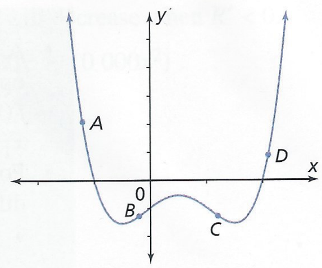
(d)



2. **(a)** The function $f(x)$ is an increasing function. Its derivative $f'(x)$ is defined for all $x \in \mathbf{R}$. Can any values of $f'(x)$ be negative? Explain.

   (b) The function $f(x)$ is a decreasing function. Its derivative $f'(x)$ is defined for all $x \in \mathbf{R}$. Can any values of $f'(x)$ be positive? Explain.

3. Determine the sign of $\frac{dy}{dx}$ at points A, B, C, and D.



4. Solve for x, $x \in \mathbf{R}$.

(a) $3x + 6 > 0$ (b) $-2x + 8 < 0$

(c) $(x - 5)(x + 2) > 0$ (d) $(3x + 2)(x - 4) < 0$

(e) $x^2 - 81 > 0$ (f) $x^2 - 10x + 24 > 0$

(g) $x^2 - x - 30 < 0$ (h) $8x^2 + 2x - 3 > 0$

(i) $2x^2 - 3x - 20 < 0$ (j) $3x^2 - 11x > 4$

5. For each function $f(x)$, determine $f'(x)$. Also determine when $f'(x) > 0$. State the intervals on which $f(x)$ is increasing.

(a) $f(x) = 2x + 10$ (b) $f(x) = -4x + 9$

(c) $f(x) = x^2 + 3$ (d) $f(x) = -2x^2 - 8$

(e) $f(x) = 4x^2 + 8x$ (f) $f(x) = -5x^2 - 20x + 3$

6. For each function $f(x)$, determine $f'(x)$. Also determine when $f'(x) < 0$. State the intervals on which $f(x)$ is decreasing.

(a) $f(x) = -3x - 12$ (b) $f(x) = 5x + 35$

(c) $f(x) = 3x^2 + 13$ (d) $f(x) = -3x^2 - 12$

(e) $f(x) = 3x^2 + 12x - 2$ (f) $f(x) = -4x^2 - 32x + 5$

B

7. Determine the intervals where each function increases and decreases.

(a) $y = 8x + 16$ (b) $y = -3x - 1$

(c) $y = 5$ (d) $y = x^2 + 4x + 1$

(e) $y = 6 - 3x^2$ (f) $y = -x^2 + 2x - 1$

(g) $y = x^3 - 12x + 15$ (h) $y = x^3 - 27x - 10$

(i) $y = -2x^3 + 6x - 2$ (j) $y = x^3 + 2$

8. Determine the intervals where each function increases and decreases.

(a) $y = 2x^4 + 10$ (b) $y = -3x^4 - 12x$

(c) $y = x^4 - 2x^2 - 1$ (d) $y = 3x^4 + 4x^3 - 12x^2$

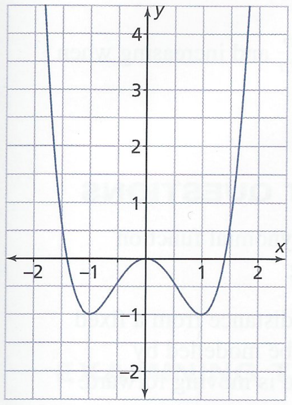
(e) $y = 2x^2 - \frac{1}{4}x^4$ (f) $y = x^4 + x^2 - 1$

9. **Knowledge and Understanding:** Determine where $g(x) = 2x^3 - 3x^2 - 12x + 15$ is increasing and where it is decreasing.

10. A plastic pop bottle holds 2 L of liquid. In an experiment, a small hole is drilled in the bottom of the bottle. The volume of liquid, V, remaining after t seconds can be modelled by $V(t) = 2 - \frac{t}{5} + \frac{t^2}{200}$, where $t \geq 0$.

(a) How long does it take for the 2 L of liquid to drain from the bottle?

(b) Verify that the volume of liquid is always decreasing until the bottle is empty.



11. **Communication:** Identify the intervals on which the function shown on the left is increasing or decreasing.

12. A slow-pitch pitcher lobs the ball toward home plate. The height of the ball in metres, h, at t seconds can be modelled by $h(t) = -4.9t^2 + 10.5t + 0.2$.

(a) When is the height of the ball increasing? decreasing?

(b) When is the velocity of the ball increasing? decreasing?

13. **Application:** The profit, P, in dollars for selling x hamburgers is modelled by $P(x) = 2.44x - \frac{x^2}{20\,000} - 5000$, where $0 \leq x \leq 35\,000$. For what quantities of hamburgers is the profit increasing? decreasing?

14. Graph f if $f'(x) < 0$ when $x < -2$ and when $x > 3$, $f'(x) > 0$ when $-2 < x < 3$, and $f(-2) = 0$ and $f(3) = 5$.

15. Graph f if $f'(x) > 0$ when $x < -3$ and when $x > 1$, $f'(x) < 0$ when $-3 < x < 1$, and $f(-3) = 4$ and $f(1) = 2$.

16. Use an example to show and verify the following: If functions f and g are increasing on an interval I, then $f + g$ must also be increasing on I.

17. The world population from 1900 to 2000 can be modelled by $P(t) = 0.0012t^3 + 0.3197t^2 + 0.2109t + 1688.951$, where P is the population in millions and t is the number of years since 1900. Did the world population ever decrease in the 20th century? Justify your answer.

18. **Thinking, Inquiry, Problem Solving:** After birth, a baby normally loses weight for a few days and then starts gaining. The table shows an infant's weight during the first two weeks.

Day	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Weight (kg)	3.14	3.03	2.95	2.80	2.77	2.76	2.79	2.84	2.93	2.95	3.01	3.14	3.32	3.49	3.68

Determine the best polynomial model for this data. Use your model to find the intervals on which the infant's weight is increasing and decreasing.

19. **Check Your Understanding:** A rock is thrown into the air from a bridge. Verify that its height decreases over the interval $1 < t < 2.2$. The height of the rock above the water, h, in metres at t seconds is modelled by $h(t) = -4.9t^2 + 9.8t + 2.1$.

C **20.** Determine the intervals in which $f(x) = |x - 2| + 3$ increases and decreases.

21. For the cubic polynomial function $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$, find conditions for a, b, c, and d to ensure that, for $-\infty < x < \infty$, f is always

(a) increasing **(b)** decreasing

22. Use calculus to prove that, for any quadratic function $f(x) = ax^2 + bx + c$,

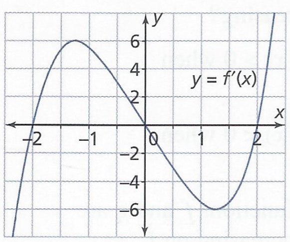
(a) if $a > 0$, then f is always decreasing when $x < -\frac{b}{2a}$ and increasing when $x > -\frac{b}{2a}$

(b) if $a < 0$, then f is always increasing when $x < -\frac{b}{2a}$ and increasing when $x > -\frac{b}{2a}$

ADDITIONAL ACHIEVEMENT CHART QUESTIONS

Knowledge and Understanding: Determine where the polynomial function $f(x) = 2x^3 - 9x^2 + 12x - 2$ is increasing and decreasing.

Application: A particle moves along a line. The particle's distance from a fixed point, s, in metres as a function of time, t, in seconds can be modelled by $s(t) = -2t^3 + 6t^2 - 3$, $t \geq 0$. Determine when the particle is moving forward.



Thinking, Inquiry, Problem Solving: Given the graph of $f'(x)$ on the left, determine where $f(x)$ is increasing and decreasing.

Communication: Suppose that you are riding on a Ferris wheel. Sketch a graph that represents your distance above the ground while you are on the Ferris wheel. When does the distance increase? When does the distance decrease?

The Chapter Problem

Trends in Post-Secondary Education

Apply what you learned in this section to answer these questions about The Chapter Problem on page 264.

CP1. Create a scatter plot using graphing technology. The independent variable is the number of years since 1980. Sketch the curve of best fit.

CP2. Using graphing technology, determine the equation of the polynomial that best models the given data.

CP3. Use the mathematical model you found to determine when the enrollment is increasing and decreasing between 1980 and 1998.