CHECK, CONSOLIDATE, COMMUNICATE

1. Show graphically that $\frac{d}{dx}(x^2 + 2x) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x)$.
2. If $h(x) = g(x) - f(x)$, where $g(x) = 4x^3$ and $f(x) = 5x^2$, does $h'(2) = g'(2) - f'(2)$? Explain.
3. Explain why any polynomial function can be differentiated term by term.

KEY IDEAS

The table summarizes the differentiation rules developed in this section.

Rule	Function Notation	Leibniz Notation
Sum Rule	If $h(x) = f(x) + g(x)$ and f and g are both differentiable, then $h'(x) = f'(x) + g'(x)$.	$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$
Difference Rule	If $h(x) = f(x) - g(x)$ and f and g are both differentiable, then $h'(x) = f'(x) - g'(x)$.	$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$
Derivative of a Polynomial	If $P(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_2x^2 + a_1x^1 + a_0$, where $n \in \mathbf{N}$, then $P'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + 2a_2x^1 + a_1$.	

3.6 Exercises

A

1. For $y = 3x^2 + 2x - 1$,
   (a) find $\frac{dy}{dx}$ from first principles
   (b) confirm your results for (a) using the rules for differentiation
2. Differentiate.
   (a) $y = 4x^2 + 5x - 2$
   (b) $y = x^3 - 5x^2 + 2x - 8$
   (c) $y = 3x + 2$
   (d) $y = -4x^{-2} + 5x - 1$
   (e) $f(x) = 8x + 3$
   (f) $f(x) = 3x^2 + 2x - 5$
   (g) $f(x) = \frac{3}{x^4} - \frac{2}{x} + 5$
   (h) $f(x) = 6x^4 - 3x^3 + 9x^2 - 5x + 8$
3. Determine the slope of the tangent line at the given point for each function.
   (a) $y = 3x + 5$ at $(2, 11)$
   (b) $y = 4x^2 - 3x + 7$ at $(-1, 14)$
   (c) $y = -2x^3$ at $(-2, 16)$
   (d) $y = 5x^4 - 4x^3 + 3x^2 - 6x + 2$ at $(0, 2)$
   (e) $y = -5x^2 + 6x - 3$ at $(0, -3)$
   (f) $y = -7x^4 - x^2 + 6x$ at $(1, -2)$

4. **(a)** Find the equation of the tangent to the curve of $y = 2x^2 - 5x - 7$ where $x = 1$.

   (b) Draw a sketch of the function and the tangent.

5. **Communication:** Express in your own words the sum and difference rules for differentiation. Use an example to illustrate each rule.

B

6. **Knowledge and Understanding:** Determine $\frac{dy}{dx}$ for the function $y = 5x^4 - 8x^3 + 3x^2 - 6x + 9$.

7. Determine the equation of the tangent to the curve of $y = x^2 - 3x + 1$ at each point.

   (a) $(-1, 5)$ **(b)** $(-2, 11)$ **(c)** $(0, 1)$ **(d)** $\left(\frac{1}{2}, -\frac{1}{4}\right)$

8. For $f(x) = 3x^2 + 8x - 5$ and $g(x) = 5x^3 + 4x^2 - 5x + 7$, show that

   (a) the derivative of the sum equals the sum of the derivatives

   (b) the derivative of the difference equals the difference between the derivatives

9. Find the equation of the tangent to each curve.

   (a) $y = x^2 - 5x + 4$ where $x = 3$

   (b) $y = 4x^2 + x - 5$ where $x = -\frac{1}{2}$

   (c) $f(x) = x^3 - 5x^2 + 6x - 7$ where $x = -1$

   (d) $f(x) = 5x^4 + x^3 - 6x$ where $x = 3$

10. Find the equations of all the tangents to the graph of $f(x) = x^2 - 4x + 25$ that pass through the origin.

11. **Application:** Liquid is flowing out of a tank. The volume, V, in litres remaining after t minutes is given by $V(t) = 1000(20 - t^2)$.

    (a) What is the initial volume of liquid in the tank?

    (b) Over the first two minutes, what is the average rate at which the tank is being emptied?

    (c) At exactly what time is this rate in effect?

    (d) How fast is the liquid leaving the tank at 3 min?

    (e) How long, to the nearest half minute, will the liquid take to drain completely from the tank?

    (f) What is the average rate, to the nearest litre per minute, at which the liquid drains?

12. A business report determines that a company's profit, P, in dollars per month can be expressed as a function of the number of items manufactured, x:

$$P(x) = -x^3 + 32x^2 + 560x - 9600, \ 0 \le x \le 40$$

    (a) Explain why the y-intercept of the graph is negative.

(b) At what rate is the profit changing when 15 items are manufactured? 35? 26?

(c) What is the profit when 15 items are manufactured? 35? 26?

(d) For what levels of production is the company profitable?

13. Kathy has diabetes. Her blood sugar level, B, one hour after an insulin injection, depends on the amount of insulin, x, in milligrams injected.

$$B(x) = -0.2x^2 + 500, 0 \leq x \leq 40$$

(a) Find $B(0)$ and $B(30)$.

(b) Find $B'(0)$ and $B'(30)$.

(c) Interpret your results.

(d) Consider the values of $B'(50)$ and $B(50)$. Comment on the significance of these values. Why are restrictions given for the original function?

14. (a) Find coordinates of the points, if any, where each function has a horizontal tangent line.

i. $f(x) = 2x - 5x^2$

ii. $f(x) = 4x^2 + 2x - 3$

iii. $f(x) = x^3 - 8x^2 + 5x + 3$

(b) Suggest a graphical interpretation for each of these points.

15. **Thinking, Inquiry, Problem Solving:** Find numbers a, b, and c so that the graph of $f(x) = ax^2 + bx + c$ has x-intercepts at (0, 0) and (8, 0), and a tangent with slope 16 where $x = 2$.

16. The population, P, of a bacteria colony at t hours can be modelled by

$$P(t) = 100 + 120t + 10t^2 + 2t^3$$

(a) What is the initial population of the bacteria colony?

(b) What is the population of the colony at 5 h?

(c) What is the growth rate of the colony at 5 h?

17. Coffee consumption in the United States can be modelled by $C(x) = 2.767\,75 + 0.084\,794\,3x - 0.008\,320\,58x^2 + 0.000\,144\,017x^3$, where C represents the number of cups consumed per day by the average adult and x represents the number of years since 1955.

(a) How many cups of coffee did the average American adult consume each day in 2000?

(b) What was the rate of change in the number of cups of coffee consumed per adult per day in 2000?

18. **Check Your Understanding**

(a) Determine $f'(3)$, where $f(x) = -6x^3 + 4x - 5x^2 + 10$.

(b) Give two interpretations of the meaning of $f'(3)$.