

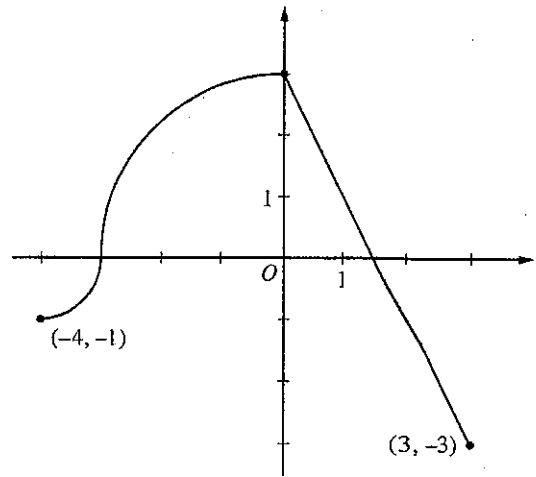
**AP[®] CALCULUS AB
2011 SCORING GUIDELINES**

Question 4

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t) dt$.

- (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
- (b) Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.
- (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
- (d) Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.



Graph of f

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t) dt = -6 - \frac{9\pi}{4}$
 $g'(x) = 2 + f(x)$
 $g'(-3) = 2 + f(-3) = 2$

3 : $\begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.
 $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

3 : $\begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

Therefore g has an absolute maximum at $x = \frac{5}{2}$.

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \leq x \leq 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

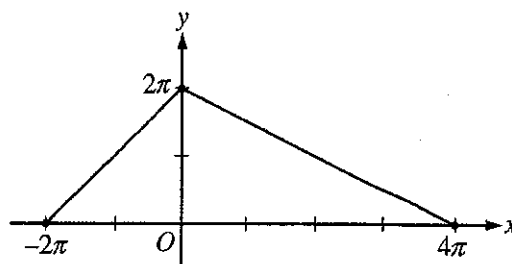
2 : $\begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

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Question 6

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.



Graph of g

- (a) Find $\int_{-2\pi}^{4\pi} f(x) dx$. Show the computations that lead to your answer.
- (b) Find all x -values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.
- (c) Let $h(x) = \int_0^{3x} g(t) dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

$$\begin{aligned} \text{(a)} \quad \int_{-2\pi}^{4\pi} f(x) dx &= \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right) \right) dx \\ &= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right) \right]_{x=-2\pi}^{x=4\pi} \\ &= 6\pi^2 \end{aligned}$$

2 : $\begin{cases} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

$$\text{(b)} \quad f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$$

4 : $\begin{cases} 1 : \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1 : g'(x) \\ 1 : x = 0 \\ 1 : x = \pi \end{cases}$

$f'(x)$ does not exist at $x = 0$.

For $-2\pi < x < 0$, $f'(x) \neq 0$.

For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

f has critical points at $x = 0$ and $x = \pi$.

$$\begin{aligned} \text{(c)} \quad h'(x) &= g(3x) \cdot 3 \\ h'\left(-\frac{\pi}{3}\right) &= 3g(-\pi) = 3\pi \end{aligned}$$

3 : $\begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$

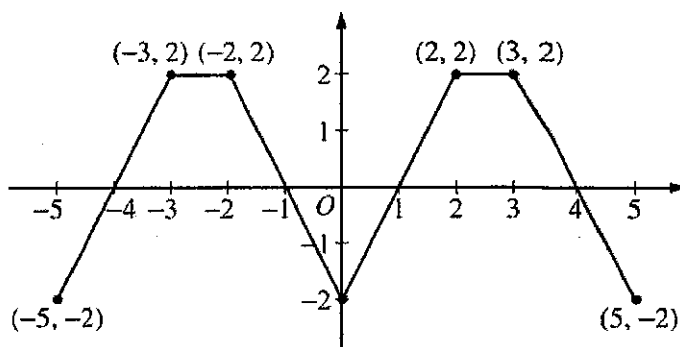
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2006 SCORING GUIDELINES

Question 3

The graph of the function f shown above consists of six line segments. Let g be the function given by

$$g(x) = \int_0^x f(t) dt.$$

- (a) Find $g(4)$, $g'(4)$, and $g''(4)$.
 (b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.



Graph of f

- (c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

(a) $g(4) = \int_0^4 f(t) dt = 3$

$$g'(4) = f(4) = 0$$

$$g''(4) = f'(4) = -2$$

- (b) g has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

$$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$$

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

- (c) $g(0) = 0$ and the function values of g increase by 2 for every increase of 5 in x .

$$g(10) = 2g(5) = 4$$

$$\begin{aligned} g(108) &= \int_0^{105} f(t) dt + \int_{105}^{108} f(t) dt \\ &= 21g(5) + g(3) = 44 \end{aligned}$$

$$g'(108) = f(108) = f(3) = 2$$

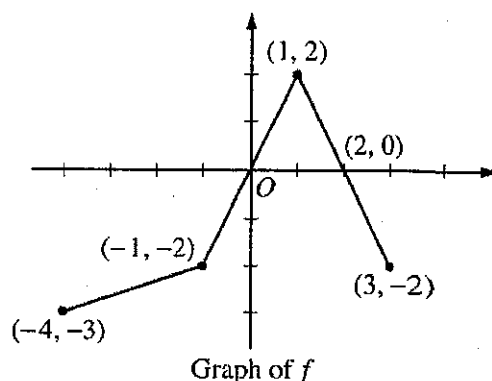
An equation for the line tangent to the graph of g at $x = 108$ is $y - 44 = 2(x - 108)$.

$$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$

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2005 SCORING GUIDELINES (Form B)

Question 4

The graph of the function f above consists of three line segments.



(a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

(a) $g(-1) = \int_{-4}^{-1} f(t) dt = -\frac{1}{2}(3)(5) = -\frac{15}{2}$
 $g'(-1) = f(-1) = -2$
 $g''(-1)$ does not exist because f is not differentiable at $x = -1$.

3 : $\begin{cases} 1 : g(-1) \\ 1 : g'(-1) \\ 1 : g''(-1) \end{cases}$

(b) $x = 1$
 $g' = f$ changes from increasing to decreasing at $x = 1$.

2 : $\begin{cases} 1 : x = 1 \text{ (only)} \\ 1 : \text{reason} \end{cases}$

(c) $x = -1, 1, 3$

2 : correct values
 $\langle -1 \rangle$ each missing or extra value

(d) h is decreasing on $[0, 2]$
 $h' = -f < 0$ when $f > 0$

2 : $\begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

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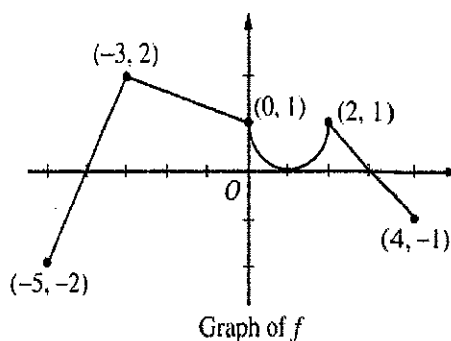
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2004 SCORING GUIDELINES**

Question 5

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function

given by $g(x) = \int_{-3}^x f(t) dt$.

- (a) Find $g(0)$ and $g'(0)$.
- (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.
- (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.
- (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.



(a) $g(0) = \int_{-3}^0 f(t) dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$
 $g'(0) = f(0) = 1$

2 : $\begin{cases} 1 : g(0) \\ 1 : g'(0) \end{cases}$

- (b) g has a relative maximum at $x = 3$.
 This is the only x -value where $g' = f$ changes from positive to negative.

2 : $\begin{cases} 1 : x = 3 \\ 1 : \text{justification} \end{cases}$

- (c) The only x -value where f changes from negative to positive is $x = -4$. The other candidates for the location of the absolute minimum value are the endpoints.

3 : $\begin{cases} 1 : \text{identifies } x = -4 \text{ as a candidate} \\ 1 : g(-4) = -1 \\ 1 : \text{justification and answer} \end{cases}$

$g(-5) = 0$

$g(-4) = \int_{-3}^{-4} f(t) dt = -1$

$g(4) = \frac{9}{2} + \left(2 - \frac{\pi}{2}\right) = \frac{13 - \pi}{2}$

So the absolute minimum value of g is -1 .

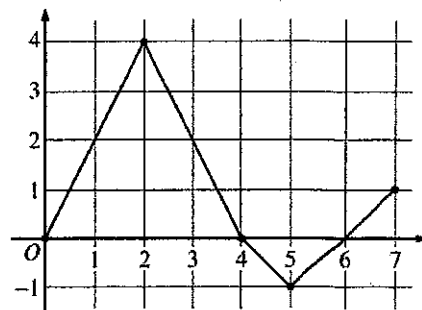
- (d) $x = -3, 1, 2$

2 : correct values
 $\langle -1 \rangle$ each missing or extra value

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2003 SCORING GUIDELINES (Form B)

Question 5

Let f be a function defined on the closed interval $[0, 7]$. The graph of f , consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t) dt$.



Graph of f

- (a) Find $g(3)$, $g'(3)$, and $g''(3)$.
- (b) Find the average rate of change of g on the interval $0 \leq x \leq 3$.
- (c) For how many values c , where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.
- (d) Find the x -coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

(a) $g(3) = \int_2^3 f(t) dt = \frac{1}{2}(4 + 2) = 3$
 $g'(3) = f(3) = 2$
 $g''(3) = f'(3) = \frac{0 - 4}{4 - 2} = -2$

3 : $\left\{ \begin{array}{l} 1 : g(3) \\ 1 : g'(3) \\ 1 : g''(3) \end{array} \right.$

(b) $\frac{g(3) - g(0)}{3} = \frac{1}{3} \int_0^3 f(t) dt$
 $= \frac{1}{3} \left(\frac{1}{2}(2)(4) + \frac{1}{2}(4 + 2) \right) = \frac{7}{3}$

2 : $\left\{ \begin{array}{l} 1 : g(3) - g(0) = \int_0^3 f(t) dt \\ 1 : \text{answer} \end{array} \right.$

(c) There are two values of c .
 We need $\frac{7}{3} = g'(c) = f(c)$

2 : $\left\{ \begin{array}{l} 1 : \text{answer of 2} \\ 1 : \text{reason} \end{array} \right.$

The graph of f intersects the line $y = \frac{7}{3}$ at two places between 0 and 3.

Note: 1/2 if answer is 1 by MVT

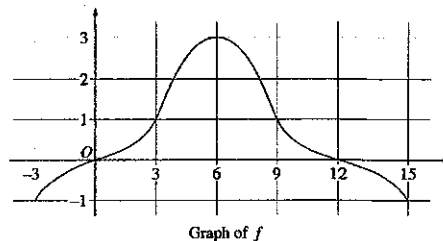
(d) $x = 2$ and $x = 5$
 because $g' = f$ changes from increasing to decreasing at $x = 2$, and from decreasing to increasing at $x = 5$.

2 : $\left\{ \begin{array}{l} 1 : x = 2 \text{ and } x = 5 \text{ only} \\ 1 : \text{justification} \\ \quad (\text{ignore discussion at } x = 4) \end{array} \right.$

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2002 SCORING GUIDELINES (Form B)

Question 4

The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let



$g(x) = 5 + \int_6^x f(t) dt$ for $-3 \leq x \leq 15$.

- (a) Find $g(6)$, $g'(6)$, and $g''(6)$.
 (b) On what intervals is g decreasing? Justify your answer.
 (c) On what intervals is the graph of g concave down? Justify your answer.
 (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t) dt$ using six subintervals of length $\Delta t = 3$.

(a) $g(6) = 5 + \int_6^6 f(t) dt = 5$
 $g'(6) = f(6) = 3$
 $g''(6) = f'(6) = 0$

3 $\left\{ \begin{array}{l} 1 : g(6) \\ 1 : g'(6) \\ 1 : g''(6) \end{array} \right.$

(b) g is decreasing on $[-3, 0]$ and $[12, 15]$ since
 $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$.

3 $\left\{ \begin{array}{l} 1 : [-3, 0] \\ 1 : [12, 15] \\ 1 : \text{justification} \end{array} \right.$

(c) The graph of g is concave down on $(6, 15)$ since
 $g' = f$ is decreasing on this interval.

2 $\left\{ \begin{array}{l} 1 : \text{interval} \\ 1 : \text{justification} \end{array} \right.$

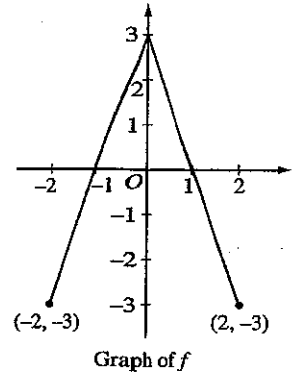
(d) $\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1)$
 $= 12$

1 : trapezoidal method

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Question 4

The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.



- (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.
- (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.
- (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.
- (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.

(a) $g(-1) = \int_0^{-1} f(t) dt = -\int_{-1}^0 f(t) dt = -\frac{3}{2}$
 $g'(-1) = f(-1) = 0$
 $g''(-1) = f'(-1) = 3$

3 $\left\{ \begin{array}{l} 1: g(-1) \\ 1: g'(-1) \\ 1: g''(-1) \end{array} \right.$

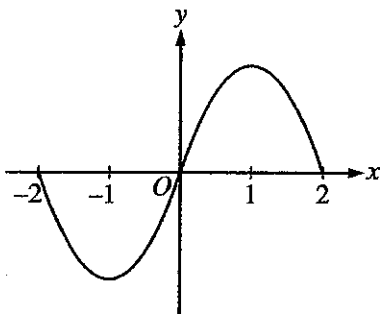
(b) g is increasing on $-1 < x < 1$ because $g'(x) = f(x) > 0$ on this interval.

2 $\left\{ \begin{array}{l} 1: \text{interval} \\ 1: \text{reason} \end{array} \right.$

(c) The graph of g is concave down on $0 < x < 2$ because $g''(x) = f'(x) < 0$ on this interval.
 or
 because $g'(x) = f(x)$ is decreasing on this interval.

2 $\left\{ \begin{array}{l} 1: \text{interval} \\ 1: \text{reason} \end{array} \right.$

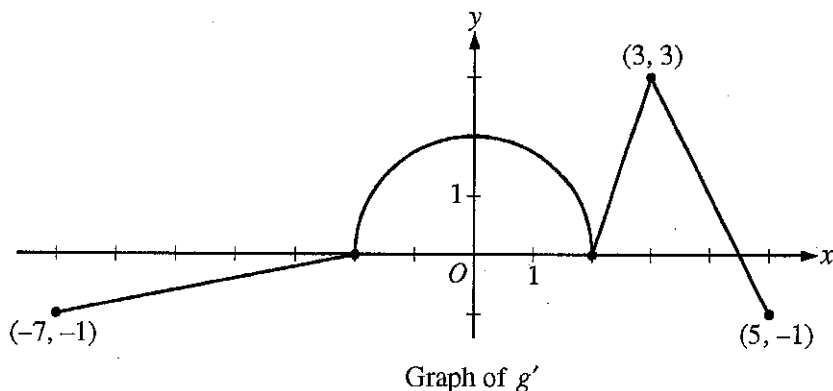
(d)



2 $\left\{ \begin{array}{l} 1: g(-2) = g(0) = g(2) = 0 \\ 1: \text{appropriate increasing/decreasing} \\ \text{and concavity behavior} \\ <-1> \text{vertical asymptote} \end{array} \right.$

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Question 5



The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g , consists of a semicircle and three line segments, as shown in the figure above.

- (a) Find $g(3)$ and $g(-2)$.
- (b) Find the x -coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.
- (c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x -coordinate of each critical point of h , where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

$$(a) \quad g(3) = 5 + \int_0^3 g'(x) \, dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$$

$$g(-2) = 5 + \int_0^{-2} g'(x) \, dx = 5 - \pi$$

$$3 : \begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$$

- (b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

$$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$$

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$

On the interval $-2 \leq x \leq 2$, $g'(x) = \sqrt{4 - x^2}$.

On this interval, $g'(x) = x$ when $x = \sqrt{2}$.

The only other solution to $g'(x) = x$ is $x = 3$.

$h'(x) = g'(x) - x > 0$ for $0 \leq x < \sqrt{2}$

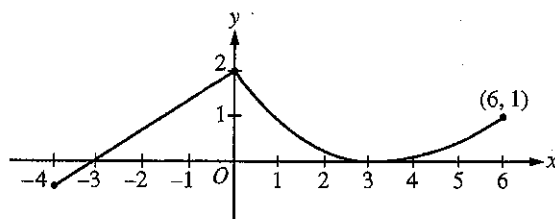
$h'(x) = g'(x) - x \leq 0$ for $\sqrt{2} < x \leq 5$

Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

$$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for } 3 \text{ with analysis} \end{cases}$$

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2009 SCORING GUIDELINES (Form B)

Question 3



Graph of f

A continuous function f is defined on the closed interval $-4 \leq x \leq 6$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

- (a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.
- (b) For how many values of a , $-4 \leq a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.
- (c) Is there a value of a , $-4 \leq a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c , $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.
- (d) The function g is defined by $g(x) = \int_0^x f(t) dt$ for $-4 \leq x \leq 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

(a)
$$\lim_{h \rightarrow 0^-} \frac{f(h) - f(0)}{h} = \frac{2}{3}$$

$$\lim_{h \rightarrow 0^+} \frac{f(h) - f(0)}{h} < 0$$

Since the one-sided limits do not agree, f is not differentiable at $x = 0$.

2 : $\left\{ \begin{array}{l} 1 : \text{sets up difference quotient at } x = 0 \\ 1 : \text{answer with justification} \end{array} \right.$

(b)
$$\frac{f(6) - f(a)}{6 - a} = 0 \text{ when } f(a) = f(6). \text{ There are two values of } a \text{ for which this is true.}$$

(c) Yes, $a = 3$. The function f is differentiable on the interval $3 < x < 6$ and continuous on $3 \leq x \leq 6$.

Also,
$$\frac{f(6) - f(3)}{6 - 3} = \frac{1 - 0}{6 - 3} = \frac{1}{3}.$$

By the Mean Value Theorem, there is a value c ,

$$3 < c < 6, \text{ such that } f'(c) = \frac{1}{3}.$$

(d)
$$g'(x) = f(x), \quad g''(x) = f'(x)$$

$$g''(x) > 0 \text{ when } f'(x) > 0$$

This is true for $-4 < x < 0$ and $3 < x < 6$.

2 : $\left\{ \begin{array}{l} 1 : \text{expression for average rate of change} \\ 1 : \text{answer with reason} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \text{answers "yes" and identifies } a = 3 \\ 1 : \text{justification} \end{array} \right.$

3 : $\left\{ \begin{array}{l} 1 : g'(x) = f(x) \\ 1 : \text{considers } g''(x) > 0 \\ 1 : \text{answer} \end{array} \right.$