Review Exercise

1. Consider any line in space that does not pass through the origin.
   a. Is it possible for this line to intersect just one coordinate axis? exactly two? all three? none at all?
   b. Is it possible for this line to intersect just one coordinate plane? exactly two? all three? none at all?

2. Find a vector equation of the line
   a. that passes through the points $(3, 9)$ and $(-4, 2)$
   b. that passes through the point $(-5, -3)$ and is parallel to the line $\vec{r} = (4, 0) + t(0, 5)$
   c. that is perpendicular to the line $2x - 5y - 6 = 0$ and passes through the point $(0, -3)$

3. Find parametric equations of the line
   a. that passes through $(-9, 8)$ with slope $-\frac{2}{3}$
   b. that passes through $(3, -2)$ and is perpendicular to the line $\vec{r} = (4, -1) + t(3, 2)$
   c. through the points $(4, 0)$ and $(0, -2)$

4. Find a vector equation of the line
   a. that passes through the points $(2, 0, -3)$ and $(-3, 2, -2)$
   b. that has an x-intercept of -7 and a z-intercept of 4
   c. that is parallel to $\frac{x-5}{4} = \frac{y+2}{-2} = \frac{z+6}{5}$ and passes through the point $(0, 6, 0)$

5. Find parametric equations of the line
   a. that is parallel to the line $\frac{x+1}{-3} = \frac{y+2}{-2} = z + 3$ and passes through the origin
   b. that passes through the point $(6, -4, 5)$ and is parallel to the y-axis
   c. that has a z-intercept of -3 and direction vector $(1, -3, 6)$

6. Find the Cartesian equation of the line
   a. that passes through the point $(-1, -2)$ and is parallel to the line $3x - 4y + 5 = 0$
   b. that passes through the point $(-7, 3)$ and is perpendicular to the line $x = 2 + t, y = -3 + 2t$
   c. that passes through the origin and is perpendicular to the line $x + 4y + 1 = 0$

7. a. Find the parametric equations of the line l that passes through the point $A(6, 4, 0)$ and is parallel to the line passing through $B(-2, 0, 4)$ and $C(3, -2, 1)$.
   b. If $(-4, m, n)$ is a point on l, find m and n.

8. Determine if the following pairs of lines are parallel and distinct, coincident, perpendicular, or none of these.
   a. $\vec{r} = (2, 3) + t(-3, 1)$ and $\vec{r} = (-1, 4) + u(6, -2)$
   b. $x = 1 + 2t, y = -3 - t$ and $x = u, y = \frac{1}{3} + 2u$
   c. $\frac{x-1}{2} = \frac{y+4}{1}, z = 1$ and $x = 4t, y = 1 + 2t, z = 6$
   d. $(x, y, z) = (1, 7, 2) + t(-1, -1, 1)$ and $(x, y, z) = (-3, 0, 1) + u(2, -2, -2)$

9. At what points does the line $\frac{x+4}{2} = \frac{y-6}{-1} = \frac{z+2}{4}$ meet the coordinate planes?

10. In the xy-plane,
    a. find the Cartesian equation of the line $\vec{r} = (2, 3) + t(-1, 5)$
    b. find a vector equation of the line $5x - 2y + 10 = 0$
    c. find a vector equation of the line $y = \frac{3}{4}x + \frac{1}{2}$

11. Given the line $\vec{r} = (12, -8, -4) + t(-3, 4, 2)$,
    a. find the intersections with the coordinate planes, if any
    b. find the intercepts with the coordinate axes, if any
    c. graph the line in an x-, y-, z-coordinate system

12. Find the direction cosines and the direction angles (to the nearest degree) of the direction vectors of the following lines.
    a. $\frac{x-3}{5} = \frac{y+6}{2} = \frac{z-1}{-1}$
    b. $x = 1 + 8t, y = 2 - t, z = 4 - 4t$
    c. $\vec{r} = (-7, 0, 0) + t(4, 1, 0)$

13. Find the intersection, if any, of
    a. the line $\vec{r} = (0, 0, 2) + t(4, 3, 4)$ and the line $\vec{r} = (-4, 1, 0) + u(-4, 1, -2)$
    b. the line $x = t, y = 1 + 2t, z = 3 - t$ and the line $x = -3, y = -6 + 2u, z = 3 - 6u$

14. Find the shortest distance between
    a. the points $(2, 1, 3)$ and $(0, -4, 7)$
    b. the point $(3, 7)$ and the line $2x - 3y = 7$
    c. the point $(4, 0, 1)$ and the line $\vec{r} = (2, -2, 1) + t(1, 2, -1)$
    d. the point $(1, 3, 2)$ and the line $\frac{x-1}{-1} = \frac{y-3}{1} = \frac{z-7}{2}$

15. Find the coordinates of the foot of the perpendicular from $Q(3, 2, 4)$ to the line $\vec{r} = (-6, -7, -3) + t(5, 3, 4)$.

Exercise 7.3

2. a. $(4, -2, 5)$ b. $(7, -2, 3)$ c. $(-1, 2, 4)$
3. a. $(4, 0, 1), (-5, 3, 4)$ b. $(4, -2, 5), (2, 3, 9)$
c. $(4, -5, -1), (7, -1, -2)$ **4.** a. $\vec{r} = (2, 4, 6) + t(1, 3, -2)$; $x = 2 + t, y = 4 + 3t, z = 6 - 2t$; $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{-2}$
b. $\vec{r} = (0, 0, -5) + t(1, -4, -1)$; $x = t, y = -4t, z = -5 - t$; $\frac{x}{1} = \frac{y}{-4} = \frac{z+5}{-1}$ c. $\vec{r} = (1, 0, 0) + t(0, 0, -1)$; $x = 1, y = 0, z = -t$ **5.** $(-20, 10, -27), (-14, 8, -17), (-8, 6, -7), (-2, 4, 3), (4, 2, 13), (10, 0, 23), (16, -2, 33)$
6. a. $P(2, 4, 2)$ b. $a = -8, b = -1$ **7.** $x = 6t, y = -1 + 4t, z = 1 + t$ **8.** $\frac{x}{6} = \frac{y}{7} = \frac{z}{-2}$ **9.** a. parallel b. neither c. same
12. $\frac{x+6}{6} = \frac{y-4}{-5} = \frac{z-2}{-2}$ **13.** b. $x = 3t, y = t, z = 2 + 6t$; $-3 \le t \le 2$ **14.** $\vec{r} = (4, 5, 5) + s(1, 5, 2)$ **15.** b. $\frac{\sqrt{66}}{6}$
c. $\sqrt{\frac{1555}{74}}$

Exercise 7.4

2. a. $(-5, -1)$ b. $(1, -2)$ **3.** a. coincident b. neither c. neither
d. parallel and distinct **4.** a. $(8, 2, 3)$ b. lines are coincident
c. skew d. parallel and distinct e. $(-1, 1, 1)$
5. a. $(-2, -3, 0)$ b. $\vec{r} = (-2, -3, 0) + s(1, -2, 1)$ **6.** $(2, 3, 1)$
7. x intercept is -4 **8.** $\left(\frac{21}{2}, -1\right)$
11. $\vec{r} = (-5, -4, 2) + t(14, -5, 2)$; $(9, -9, 4)$ **12.** $(2, -1, -1), (1, 2, 1)$, No
13. $\vec{r} = s(17, -15, -20)$ **14.** a. $\left(\frac{-AC}{A^2+B^2}, \frac{-BC}{A^2+B^2}\right)$
b. $\frac{|C|}{\sqrt{A^2+B^2}}$ **15.** $(0, 1, 2), (1, 1, 1)$ **16.** a. $\frac{1}{\sqrt{3}}$ b. 6

Review Exercise

2. a. $\vec{r} = (3, 9) + t(1, 1)$ b. $\vec{r} = (-5, -3) + t(1, 0)$
c. $\vec{r} = (0, -3) + t(2, -5)$ **3.** a. $x = -9 + 3t, y = 8 - 2t$
b. $x = 3 + 2s, y = -2 - 3s$ c. $x = 4 + 2t, y = t$
4. a. $\vec{r} = (2, 0, -3) + t(5, -2, -1)$ b. $\vec{r} = (0, 0, 4) + t(7, 0, 4)$
c. $\vec{r} = (0, 6, 0) + t(4, -2, 5)$ **5.** a. $x = 3t, y = 2t, z = -t$
b. $x = 6, y = -4 + t, z = 5$ c. $x = t, y = -3t, z = -3 + 6t$
6. a. $3x - 4y - 5 = 0$ b. $x + 2y + 1 = 0$ c. $4x - y = 0$
7. a. $x = 6 + 5t, y = 4 - 2t, z = -3t$ b. $m = 8, n = -2$
8. a. coincident b. perpendicular c. parallel and distinct
d. parallel and distinct **9.** $\left(-3, \frac{11}{2}, 0\right), (8, 0, 22), (0, 4, 6)$
10. a. $5x + y - 13 = 0$ b. $\vec{r} = (0, 5) + t(2, 5)$
c. $\vec{r} = (2, 2) + t(4, 3)$ **11.** a. $(6, 0, 0), (0, 8, 4)$
b. x intercept is 6 **12.** a. $\cos\alpha = \frac{5}{\sqrt{30}}, \cos\beta = \frac{2}{\sqrt{30}}, \cos\gamma = \frac{-1}{\sqrt{30}}$; $\alpha \cong 24°, \beta \cong 69°, \gamma \cong 101°$
b. $\cos\alpha = \frac{8}{9}, \cos\beta = \frac{-1}{9}, \cos\gamma = \frac{-4}{9}$; $\alpha \cong 27°, \beta \cong 96°, \gamma \cong 116°$ c. $\cos\alpha = \frac{4}{\sqrt{17}}, \cos\beta = \frac{1}{\sqrt{17}}, \cos\gamma = 0$; $\alpha \cong 14°, \beta \cong 76°, \gamma = 90°$ **13.** a. $(0, 0, 2)$ **14.** a. $3\sqrt{5}$
b. $\frac{22\sqrt{13}}{13}$ c. $\sqrt{2}$ d. $\frac{5\sqrt{3}}{3}$ **15.** $(4, -1, 5)$

Chapter 7 Test

1. a. $\vec{r} = (9, 2) + t(3, -1)$ b. $x = 9 + 3t, y = 2 - t$
c. $\frac{x-9}{3} = \frac{y-2}{-1}$ d. $x + 3y - 15 = 0$ **2.** $3x + 2y - 2 = 0$
3. $(-2, 2, 0), (0, 3, -1)$ **4.** $3\sqrt{2}$ **5.** $\vec{r} = (1, -1, \sqrt{2})t$; $\vec{r} = (-1, -1, \sqrt{2})t$ **6.** $(8, 2, 3)$ **7.** b. $P_1(-10, 1, 2)$
c. $P_2(-1, -2, -3)$

CHAPTER 8

Exercise 8.1

2. a. $(1, 0, 0), (4, 0, -3)$ b. y component is 0
3. a. $(-3, 5, 2), (-6, 1, 2)$ b. $(5, -5, 3), (1, 6, -2)$
c. $(4, -2, 1), (-1, 5, 2)$ **4.** a. $(9, 4, -3), (7, 4, 4)$
b. $(1, 1, 2), (1, 1, -2)$ c. $(3, -2, -2), (9, -1, -1)$
d. $(5, 0, 1), (-3, 0, 2)$ **5.** a. $x = -4 + 5s - 4t$ $y = -6 + 2s - 6t$, $z = 3 + 3s + 3t$
b. $x = 3t, y = 2s, z = 1$ c. $x = s, y = 0, z = t$
6. a. $\vec{r} = (-4, -1, 3) + s(1, 3, 4) + t(3, -4, -1)$
b. $\vec{r} = (0, 4, 0) + s(7, 0, 0) + t(0, 0, -2)$
c. $\vec{r} = s(1, 0, 0) + t(0, 0, 1)$
7. a. $\vec{r} = (-4, 5, 1) + s(-3, -5, 3) + t(2, -1, -5)$
b. $\vec{r} = (4, 7, 3) + s(1, 4, 3) + t(-1, -1, 3)$
c. $\vec{r} = (8, 3, 5) + s(5, 2, -3) + t(11, -1, -1)$
d. $\vec{r} = (0, 1, 3) + s(2, 1, -2) + t(4, -4, 7)$
e. $\vec{r} = (2, 6, -5) + s(5, 5, -1) + t(4, -8, 7)$
8. a. $x = 7 + 4s - 3t, y = -5 - s + 4t, \quad z = 2 + s + 4t$
b. $x = 5 + 2s + 4t, y = 4 - 2t, z = 2 - 9s + t$
c. $x = 8 + 5s + 2t, y = 3 - 2s + 2t, z = 5 + 11s - 5t$
d. $x = 3 + s + 3t, y = 2 - 2s - 2t, z = 2 + 4s + 2t$
e. $x = 2 + 5s + 4t, y = 6 + 5s - 8t, z = -5 - s + 7t$
9. a. $\vec{r} = (6, 4, 2) + s(0, 1, 0) + t(0, 0, 1)$
b. $\vec{r} = s(1, 1, 1) + t(8, -1, -1)$ c. $\vec{r} = s(1, 0, 0) + t(1, 4, 7)$
10. a. the three points are collinear b. the point is on the line
11. $\vec{r} = (7, 0, -7) + s(0, 0, 1) + t(1, 2, -1)$; $x = 7 + t, y = 2t, z = -7 + s - t$ **13.** b. All points in and on the parallelogram whose vertices have position vectors $\vec{a}$, $\vec{b}$, $-\vec{a} + \vec{b} + \vec{c}$ and $\vec{c}$
14. b. all points on and between the parallel lines

Exercise 8.2

1. a. $7x + y - z - 18 = 0$ b. $x - 5 = 0$ c. $2x + 3z + 6 = 0$
d. $2x - y + 4z = 0$ **2.** a. $y + 2 = 0$ b. $z - 3 = 0$
c. $x - y - 2z + 3 = 0$ **3.** a. $Ax + By + Cz = 0$ b. $D = 0$
5. a. $12x + 8y + 13z = 0$ b. $3x - 8y + z - 15 = 0$ c. $x - 2 = 0$
d. $3x + 10y - 4z + 4 = 0$ e. $x - 2 = 0$
f. $12x + 8y + 13z = 0$; a and f, c and e are coincident
6. a. $4x - 13y - 20z + 30 = 0$ b. $9x - 6y - 2z + 22 = 0$
7. a. $11x + 8y - 2z - 21 = 0$ b. $x + 3y + z = 0$ c. $y - 1 = 0$
d. $6x - 2y + 5z = 0$ **8.** $y + 2z = 0$
9. $10x + 11y - 10z - 50 = 0$ **10.** a. parallel and distinct
b. neither c. coincident d. coincident
11. a. $\vec{r} = (0, -24, 0) + s(1, 2, 0) + t(0, 3, 1)$
b. $\vec{r} = (0, 0, 3) + s(5, 0, 3) + t(0, 1, 0)$
12. a. parallel to an on the plane b. parallel to the plane, not on
c. not parallel **13.** a. 17° b. 90° **15.** a. $6x - 4y - 4z - 3 = 0$
b. a plane passing through the mid point of AB and having normal $\overrightarrow{AB}$. **16.** b. $38x + 33y + 111z - 103 = 0$
17. $x = 3t, y = -2t, z = 0$ **18.** $|D|$ will be the distance from the origin to the plane **20.** $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Review Exercise

1. a. Can a plane be perpendicular to the x-axis and contain the line $x = z, y = 0$? Explain.

   b. Can a plane be parallel to the yz-coordinate plane and contain the point $(-4, 0, 5)$? Explain.

2. Find vector and parametric equations of the plane

   a. that passes through the point $(-1, -1, 2)$ and is parallel to the plane $\vec{r} = (2, -1, 0) + s(5, 4, 2) + t(0, 0, 1)$

   b. that passes through the points $(1, 1, 0)$ and $(-2, 0, 3)$ and is parallel to the y-axis

   c. that has intercepts $x = -2$, $y = -3$, and $z = 4$

   d. that contains the point $(1, 1, 1)$ and the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

   e. that contains the two intersecting lines $\vec{r} = (3, -1, 2) + s(4, 0, 1)$ and $\vec{r} = (3, -1, 2) + t(4, 0, 2)$

3. Find the scalar equation for the plane

   a. that passes through the point $(1, 7, 9)$ and has normal $\vec{n} = (1, 3, 5)$

   b. that passes through the points $(3, 2, 3)$, $(-4, 1, 2)$, and $(-1, 3, 2)$

   c. that passes through the point $(0, 0, 6)$ and is parallel to the plane $y + z = 5$

   d. that contains the point $(3, -3, 0)$ and the line $x = 2, y = 3 + t, z = -4 - 2t$

   e. that contains the line $\vec{r} = (2, 1, 7) + s(0, 1, 0)$ and is parallel to the line $\vec{r} = (3, 0, 4) + t(2, -1, 0)$

   f. that contains the points $(6, 1, 0)$ and $(3, 0, 2)$, and is parallel to the z-axis

4. For what value of k, if any, will the planes $3x + ky + z - 6 = 0$ and $6x + (1 - k)y + 2z - 9 = 0$ be

   a. parallel?

   b. perpendicular?

5. Find the scalar equation of the plane that contains the parallel and distinct lines $x = 1, \frac{y - 3}{4} = \frac{z}{2}$ and $x = 5, \frac{y + 5}{2} = \frac{z - 3}{1}$.

6. Find a vector equation of the plane that contains the origin and the point $(2, -3, 2)$ and is perpendicular to the plane $x + 2y - z + 3 = 0$.

7. Find the scalar equation of the plane that passes through the point $(1, 2, 3)$ and is parallel to the vectors $6\hat{k}$ and $\hat{i} + 2\hat{j} - 3\hat{k}$.

8. A line that passes through the origin intersects a plane at the point $(1, -3, 2)$. If the line is perpendicular to the plane, find the scalar equation of the plane.

9. Find the scalar equation of the plane that contains the intersecting lines $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{-1}$ and $\frac{x-1}{-1} = \frac{y-1}{5} = \frac{z-1}{4}$.

10. Explain why the point $(2, 21, 8)$ and the line $\vec{r} = (-4, -3, -1) + t(2, 8, 3)$ do not determine a plane.

11. Find the distance between
    a. the point $(7, 7, -7)$ and the plane $6y - z + 5 = 0$
    b. the point $(3, 2, 1)$ and the plane $3x+2y+z=10$
    c. the line $\vec{r} = (1, 3, 2) + t(1, 2, -1)$ and the plane $y + 2z = 5$
    d. the planes $x + 2y - 5z - 10 = 0$ and $2x + 4y - 10z - 17 = 0$

12. Find the distance from the point $(1, -2, -2)$ to the plane having an x-intercept of -1, a y-intercept of 2, and a z-intercept of 3.

13. A normal to the plane $4x - 2y + 5z - 9 = 0$ passes through the origin. At what point does this normal intersect the plane?

14. Determine where the plane $4x + 5y - z + 20 = 0$ meets the coordinate axes, and graph the plane.

15. Graph the following planes in an xyz-coordinate system:
    a. $2x + y + z - 3 = 0$
    b. $3y - 4z + 24 = 0$
    c. $3z + 9 = 0$
    d. $\vec{r} = (4, -5, 0) + s(-12, 9, 8) + t(8, -7, -8)$

16. Show that the line $x = -5 - 3t$, $y = 3 - 4t$, $z = 1 + 5t$ lies in the plane $2x + y + 2z + 5 = 0$.

17. For what values of k will the planes $2x - 6y + 4z + 3 = 0$ and $3x - 9y + 6z + k = 0$
    a. not intersect?
    b. intersect in a line?
    c. intersect in a plane?

18. A plane passes through the points $(1, 0, 2)$ and $(-1, 1, 0)$ and is parallel to the vector $(-1, 1, 1)$.
    a. Find the scalar equation of the plane.
    b. Find the equation of the line through the point $Q(0, 3, 3)$ that is perpendicular to the plane.
    c. Find the point at which the perpendicular through Q intersects the plane.
    d. Use a distance formula to check your answer to part **c.**

19. Find the equation of the plane that passes through the point $(3, 0, -4)$ and is perpendicular to the line of intersection of the planes $x + 2y - 7z - 3 = 0$ and $x - 5y + 4z - 1 = 0$.

20. Let l be the line of intersection of the two planes $x + y + z - 1 = 0$ and $2x - 3y - z + 2 = 0$.
    a. Find the scalar equation of the plane that contains the line l and passes through the origin.
    b. Show that the plane found in part **a** makes an angle of $60°$ with the plane $x - z = 0$.

21. Are the two planes $\vec{r} = (4, 0, 3) + t(-8, 1, -9) + u(-1, 5, 7)$ and $\vec{r} = (-14, 12, -1) + p(1, 1, 3) + q(-2, 1, -1)$ parallel, coincident, or neither?

22. Solve each of the following systems of equations. Give a geometrical interpretation of each system and its solution.

    a. $x + 5y - 8 = 0$
       $5x - 7y + 8 = 0$

    b. $2x - 2y + 4z = 5$
       $x - y + 2z = 2$

    c. $3x + 2y - 4z + 1 = 0$
       $2x - y - z + 3 = 0$

    d. $x + 2y - 3z = 11$
       $2x + y = 7$
       $3x + 6y - 8z = 32$

    e. $x - y + 3z = 4$
       $x + y + 2z = 2$
       $3x + y + 7z = 9$

    f. $x + 3y + 3z = 8$
       $x - y + 3z = 4$
       $2x + 6y + 6z = 16$

    g. $x + 2y + z = -3$
       $x + 7y + 4z = -13$
       $2x - y - z = 4$

    h. $3x - 3z = 12$
       $2x - 2z = 8$
       $x - z = 4$

    i. $x + y + z = -3$
       $x + 2y + 2z = -4$
       $2x + 2y + 2z = -5$

Exercise 8.3

1. a. $(4, 6, -2)$ **b.** $(1, 1, 2)$ **c.** no intersection
d. $(x, y, z) = (2 - t, 14 - t, 1 + t)$ **e.** $(5, 15, -5)$
2. a. yes **b.** no **3. a.** $(2, 0, 0)$ **b.** $(0, -3, 0)$ **c.** $\left(0, 0, \frac{-6}{7}\right)$
4. a. **(i)** $\left(\frac{28}{27}, 0, 0\right)$ **(ii)** $\left(0, \frac{56}{55}, 0\right)$ **(iii)** $\left(0, 0, \frac{-7}{2}\right)$
b. **(i)** $\vec{r} = (-1, 2, 0) + k(-55, 54, 0)$
(ii) $\vec{r} = (0, 8, 24) + p(0, 16, 55)$
(iii) $\vec{r} = (4, 0, 10) + u(8, 0, 27)$ **5. a.** $(9, 14, 0)$
b. $\left(\frac{-3}{2}, 0, \frac{7}{2}\right)$ **c.** $(0, 2, 3)$ **6. a.** one point **b.** infinite number of points **c.** no points, one point, or an infinite number of points
7. $\left(\frac{3}{2}, -1, \frac{7}{2}\right)$ **8.** $\left(\frac{14}{5}, \frac{-2}{5}, -6\right)$ **9. a.** 1, 4, 3 **b.** 5, $\frac{-5}{2}$, -5
c. -4, 8, -8 **d.** 4, -16, 8 **10. a.** x-intercept is 4, y-intercept is 4; intersection with: xy plane is $\vec{r} = (u, 4 - u, 0)$, xz plane is $\vec{r} = (4, 0, s)$, yz plane is $\vec{r} = (0, 4, t)$ **b.** x-intercept is 3; intersection with: xy plane is $\vec{r} = (4, t, 0)$, xz plane is $\vec{r} = (4, 0, u)$
c. y-intercept is $-\frac{1}{2}$; intersection with: xy plane is $\vec{r} = \left(t, \frac{-1}{2}, 0\right)$, yz plane is $\vec{r} = \left(0, \frac{-1}{2}, s\right)$
d. x-intercept is 2, z-intercept is 6; intersection with: xy plane is $\vec{r} = (2, t, 0)$, xz plane is $\vec{r} = (u, 0, 6 - 3u)$, yz plane is $\vec{r} = (0, s, 6)$
e. y-intercept is 0, z-intercept is 0; intersection with: xy and xz plane is $\vec{r} = (t, 0, 0)$, yz plane is $\vec{r} = (0, 2u, u)$, **f.** x, y, is and z-intercepts are each 0; intersection with: xy plane is $\vec{r} = (t, -t, 0)$, xz plane is $\vec{r} = (s, 0, s)$, yz plane is $\vec{r} = (0, u, u)$
11. a. no value **b.** $k = 9$ **c.** $k \neq 9$

Exercise 8.4

2. a. yes **b.** no **c.** no **d.** yes **3. a.** $x = 7 + 5t$, $y = -3 - 2t$, $z = t$ **b.** parallel **c.** $x = 8 - 7t$, $y = t$, $z = 11 - 10t$
d. $x = 0$, $y = 1 - t$, $z = t$ **e.** parallel
4. a. $\left[\begin{array}{ccc|c} 3 & -7 & 1 & 12 \\ 1 & 1 & -2 & -3 \end{array}\right]$ **b.** $\left[\begin{array}{ccc|c} -4 & 3 & 2 & 4 \\ 0 & 2 & -5 & 5 \end{array}\right]$
c. $\left[\begin{array}{ccc|c} 1 & 0 & 4 & 16 \\ 0 & 1 & -8 & -2 \end{array}\right]$ **d.** $\left[\begin{array}{ccc|c} 6 & 5 & -2 & 4 \\ -2 & 5 & 3 & -4 \end{array}\right]$
5. a. $x + 4z = 9$, $y - 6z = 4$
b. $8x - 2y + 3z = -6$, $2x - 6y - 6z = 9$
c. $5x - 10y = 8$, $3y - 4z = 6$ **d.** $x + 4z = 0$, $y + 9z = 0$
6. a. $\vec{r} = (10, -3, 0) + t(-15, 4, 1)$
b. $\vec{r} = \left(\frac{5}{9}, \frac{-10}{9}, 0\right) + t(-3, 0, 1)$
c. $\vec{r} = \left(\frac{13}{4}, 0, \frac{1}{4}\right) + t(-4, 1, 0)$
d. $\vec{r} = (0, 0, -2) + t(2, 1, 0)$
e. $\vec{r} = (7, -8, 0) + t(0, 3, 1)$ **f.** $\vec{r} = (0, 0, -3) + t(1, 0, 2)$
7. a. 3 planes intersect at the point $\left(\frac{-253}{30}, \frac{106}{15}, \frac{154}{15}\right)$
b. no solution, the 3 lines are not concurrent
c. the 4 planes have no common intersection
8. b. $4x + 5y - 14z = 0$ **c.** $8x - 8y - 12z + 15 = 0$
9. $x - z - 14 = 0$

Exercise 8.5

2. a. coplanar **b.** coplanar **c.** coplanar **d.** collinear
3. a. $\left(4, \frac{1}{2}, -3\right)$ **b.** $(0, 2, 0)$ **c.** $(-1, 1, -1)$ **4.** $\left(\frac{47}{5}, \frac{-27}{5}, -5\right)$
5. a. $\left[\begin{array}{ccc|c} 5 & -2 & 1 & 5 \\ 3 & 1 & -5 & 12 \\ 1 & -5 & 2 & -3 \end{array}\right]$ **b.** $\left[\begin{array}{ccc|c} -2 & 1 & -3 & 0 \\ 1 & 5 & 0 & 8 \\ 0 & 3 & 2 & -6 \end{array}\right]$
c. $\left[\begin{array}{ccc|c} 0 & 4 & -3 & 12 \\ 2 & 5 & 0 & 15 \\ 4 & 0 & 6 & 10 \end{array}\right]$ **6. a.** $x = 8$, $y = -6$, $z = 3$
b. $x - 6z = 4$, $y + 5z = -5$, $0z = 0$ **c.** $x = 0$, $y = 0$, $0z = 1$
7. $\left(\frac{27}{4}, \frac{-15}{4}, \frac{-25}{4}\right)$ **8. a.** unique solution point $(x, y, z) = (2, 3, 4)$
b. no solution, 3 distinct parallel planes **c.** infinite number of solutions, the planes intersect in the line with equation $(x, y, z) = (7 - t, t, 2)$ **d.** Infinite number of solutions, the 3 planes are coincident, $x - 2y - 3z = 1$ **e.** no solution, 2 of the planes are parallel and distinct **f.** no solution, 2 planes are coincident, and the third is parallel and distinct **g.** infinite number of solutions intersecting in the line $(x, y, z) = (6 + t, -1 - t, 2t)$ **h.** no solution, planes form a triangular prism **i.** unique solution, the origin $(0, 0, 0)$ **9.** $\frac{-7}{19}$

Review Exercise

2. a. $\vec{r} = (-1, -1, 2) + s(5, 4, 2) + t(0, 0, 1)$; $x = -1 + 5s$, $y = -1 + 4s$, $z = 2 + 2s + t$
b. $\vec{r} = (1, 1, 0) + s(0, 1, 0) + t(3, 1, -3)$; $x = 1 + 3t$, $y = 1 + s + t$, $z = -3t$
c. $\vec{r} = (0, 0, 4) + s(2, -3, 0) + t(1, 0, 2)$; $x = 2s + t$, $y = -3s$, $z = 4 + 2t$
d. $\vec{r} = s(1, 1, 1) + t(3, 4, 5)$; $x = s + 3t$, $y = s + 4t$, $z = s + 5t$
e. $\vec{r} = (3, -1, 2) + s(4, 0, 1) + t(4, 0, 2)$ $x = 3 + 4s + 4t$, $y = -1$, $z = 2 + s + 2t$ **3. a.** $x + 3y + 5z - 67 = 0$
b. $2x - 3y - 11z + 33 = 0$ **c.** $y + z - 6 = 0$
d. $8x + 2y + z - 18 = 0$ **e.** $z - 7 = 0$ **f.** $x - 3y - 3 = 0$
4. a. $\frac{1}{3}$ **b.** $k = 5$ or $k = -4$ **5.** $7x + 2y - 4z - 13 = 0$
6. $\vec{r} = s(1, 2, -1) + t(2, -3, 2)$ **7.** $2x - y = 0$
8. $x - 3y + 2z - 14 = 0$ **9.** $17x - 7y + 13z - 23 = 0$
11. a. $\frac{54}{\sqrt{37}}$ **b.** $\frac{4}{\sqrt{14}}$ **c.** $\frac{2}{\sqrt{5}}$ **d.** $\frac{3}{2\sqrt{30}}$ **12.** $\frac{22}{7}$ **13.** $\left(\frac{4}{5}, \frac{-2}{5}, 1\right)$
14. $(-5, 0, 0)$, $(0, -4, 0)$, $(0, 0, 20)$ **17. a.** $k \neq \frac{9}{2}$
b. will never intersect in a line **c.** $k = \frac{9}{2}$
18. a. $3x + 4y - z - 1 = 0$ **b.** $\vec{r} = (0, 3, 3) + t(3, 4, -1)$
c. $\left(\frac{-12}{13}, \frac{23}{13}, \frac{43}{13}\right)$ **19.** $27x + 11y + 7z - 53 = 0$
20. a. $4x - y + z = 0$ **21.** coincident **22. a.** in R^2 2 lines intersect in the point $\left(\frac{1}{2}, \frac{3}{2}\right)$, in R^3 2 planes intersecting in the line $\vec{r} = \left(\frac{1}{2}, \frac{3}{2}, t\right)$ **b.** no solution, 2 parallel planes
c. line $\vec{r} = (-1, 1, 0) + t(6, 5, 7)$ **d.** point $(2, 3, -1)$
e. no solution, triangular prism
f. line $\vec{r} = (5, 1, 0) + t(-3, 0, 1)$
g. line $\vec{r} = (1, 0, -5) + t(1, -3, 5)$
h. planes coincident with $x - z = 4$ **i.** no solution, 2 planes are parallel and distinct

Chapter 8 Test

1. a. planes are perpendicular and intersect in a line **b.** planes are parallel **c.** planes are parallel **2. a.** line is parallel to the plane, no solution **b.** intersects the plane at $(2, 2, 0)$ **4. a.** $(-5, 0, 0)$
b. $\vec{r} = (0, 0, 5) + t(1, 0, 1)$ **5.** $4x - 4y + 7z = 0$