1 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study | ***Aid***

- See Lesson 1.1, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 1 and 2.

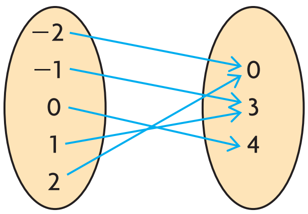
Q: How can you determine whether a relation is a function?

A1: For a relation to be a function, there must be only one value of the dependent variable for each value of the independent variable.

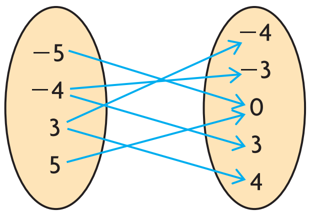
If the relation is described by a list of ordered pairs, you can see if any first elements appear more than once. If they do, the relation is not a function. For example, the relation $\{(-2, 0), (-1, 3), (0, 4), (1, 3), (2, 0)\}$ is a function; but the relation $\{(-5, 0), (-4, 3), (-4, -3), (3, -4), (3, 4), (5, 0)\}$ is not, because -4 and 3 each appear more than once as first elements.

A2: If the relation is shown in a mapping diagram, you can look at the arrows. If more than one arrow goes from an element of the domain (on the left) to an element of the range (on the right), then the relation is not a function. For example, diagram A shows a function but diagram B does not.

A

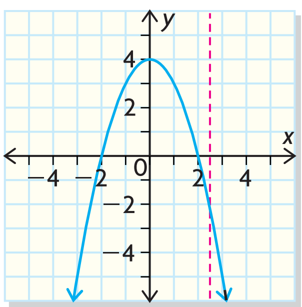


B

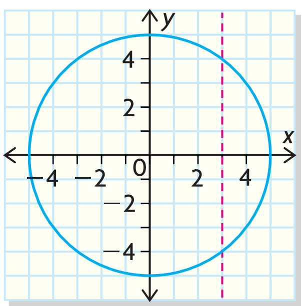


A3: If you have the graph of the relation, you can use the vertical-line test. If you can draw a vertical line that crosses the graph in more than one place, then an element in the domain corresponds to two elements in the range, so the relation is not a function. For example, graph A shows a function but graph B does not.

A



B



A4: If you have the equation of the relation, you can substitute numbers for x to see how many y-values correspond to each x-value. If a single x-value produces more than one corresponding y-value, the equation does not represent a function. For example, the equation $y = 4 - x^2$ is the equation of a function because you would get only one answer for y by putting a number in for x. The equation $x^2 + y^2 = 25$ does *not* represent a function because there are two values for y when x is any number between -5 and 5.

Study | ***Aid***

- See Lesson 1.2, Examples 1, 3, and 4.
- Try Mid-Chapter Review Questions 3 and 4.

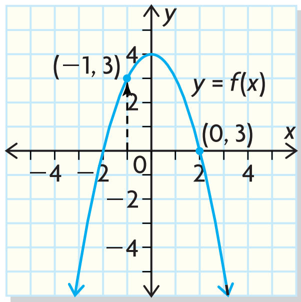
Q: What does function notation mean and why is it useful?

A: When a relation is a function, you can use function notation to write the equation. For example, you can write the equation $y = 4 - x^2$ in function notation as $f(x) = 4 - x^2$. f is a name for the function and $f(a)$ is the value of y or output when the input is $x = a$. The equation $f(-1) = 3$ means "When $x = -1$, $y = 3$;" in other words, the point $(-1, 3)$ belongs to the function.

To evaluate $f(-1)$, substitute -1 for x in the function equation:

$$\begin{aligned} f(-1) &= 4 - (-1)^2 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

Or you can read the value from a graph.



Function notation is useful because writing $f(x) = 3$ gives more information about the function—you know that the independent variable is x—than writing $y = 3$. Also, you can work with more than one function at a time by giving each function a different name. You can choose meaningful names, such as $v(t)$ to describe velocity as a function of time, t, or $C(n)$ to describe the cost of producing n items.

Q: How can you determine the domain and range of a function?

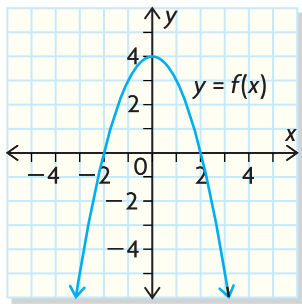
A: The domain of a function is the set of input values for which the function is defined. The range is the set of output values that correspond to the input values. Set notation can be used to describe the domain and range of a function.

Study *Aid*

- See Lesson 1.4, Examples 2 and 3.
- Try Mid-Chapter Review Questions 6, 7, and 8.

If you have the graph of a function, you can see the domain and range, as in the following examples:

A



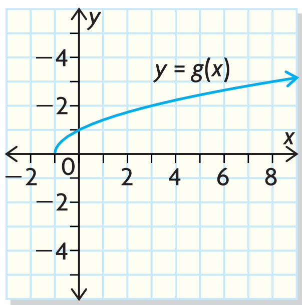
Because graph A goes on forever in both the positive and negative x direction, x can be any real number.

Because this function has a maximum value at the vertex, y cannot have a value greater than this maximum value.

You can express these facts in set notation:

$$\text{Domain} = \{x \in \mathbf{R}\}; \ \text{Range} = \{y \in \mathbf{R} \mid y \leq 4\}$$

B



Graph B starts at the point $(-1, 0)$ and continues forever in the positive x direction and positive y direction. So x can be any real number greater than or equal to -1 and y can be any real number greater than or equal to 0.

$$\text{Domain} = \{x \in \mathbf{R} \mid x \geq -1\}; \ \text{Range} = \{y \in \mathbf{R} \mid y \geq 0\}$$

You can also determine the domain and range from the equation of a function. For example, if $f(x) = 4 - x^2$, then any value of x will work in this equation, so $x \in \mathbf{R}$. Also, because x^2 is always positive or zero, $f(x)$ is always less than or equal to 4.

$$\text{Domain} = \{x \in \mathbf{R}\}; \ \text{Range} = \{y \in \mathbf{R} \mid y \leq 4\}$$

If $g(x) = \sqrt{x + 1}$, then x cannot be less than -1, or the number inside the square root sign would be negative. Also, the square root sign refers to the positive square root, so $g(x)$ is always positive or zero.

$$\text{Domain} = \{x \in \mathbf{R} \mid x \geq -1\}; \ \text{Range} = \{y \in \mathbf{R} \mid y \geq 0\}$$

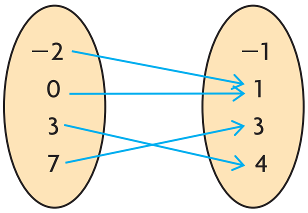
PRACTICE Questions

Lesson 1.1

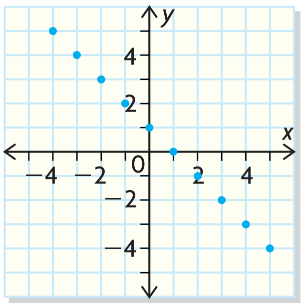
1. Determine which relations are functions. For those which are, explain why.

a) $\{(1, 2), (2, 3), (2, 4), (4, 5)\}$

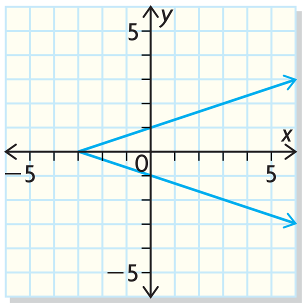
b)



c)



d)



e) $y = -(x - 3)^2 + 5$

f) $y = \sqrt{x - 4}$

2. Use numeric and graphical representations to show that $x^2 + y = 4$ is a function but $x^2 + y^2 = 4$ is not a function.

Lesson 1.2

3. a) Graph the function $f(x) = -2(x + 1)^2 + 3$.

b) Evaluate $f(-3)$.

c) What does $f(-3)$ represent on the graph of f?

d) Use the equation to determine **i)** $f(1) - f(0)$, **ii)** $3f(2) - 5$, and **iii)** $f(2 - x)$.

4. A teacher asked her students to think of a number, multiply it by 5, and subtract the product from 20. Then she asked them to multiply the resulting difference by the number they first thought of.

a) Use function notation to express the final answer in terms of the original number.

b) Determine the outputs for the input numbers 1, -1, and 7.

c) Determine the maximum result possible.

Lesson 1.3

5. Graph each function and state its domain and range.

a) $f(x) = x^2$

b) $f(x) = \frac{1}{x}$

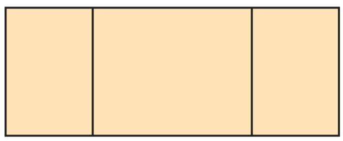
c) $f(x) = \sqrt{x}$

d) $f(x) = |x|$

Lesson 1.4

6. Determine the domain and range of each relation in question 1.

7. A farmer has 600 m of fencing to enclose a rectangular area and divide it into three sections as shown.



a) Write an equation to express the total area enclosed as a function of the width.

b) Determine the domain and range of this area function.

c) Determine the dimensions that give the maximum area.

8. Determine the domain and range for each.

a) A parabola has a vertex at $(-2, 5)$, and $y = 5$ is its maximum value.

b) A parabola has a vertex at $(3, 4)$, and $y = 4$ is its minimum value.

c) A circle has a centre at $(0, 0)$ and a radius of 7.

d) A circle has a centre at $(2, 5)$ and a radius of 4.