6 Chapter Review

FREQUENTLY ASKED Questions

Q: How do you use transformations to determine the domain and range of a sinusoidal function?

A: The domain of a sinusoidal function is $\{x \in \mathbf{R}\}$. A restriction in the domain can occur when you consider the real-world situation you are trying to model.

To determine the range, you must determine the equation of the axis, based on the vertical translation. You then determine the amplitude, based on the vertical stretch or compression. Determine the equation of the axis, and then go above and below that value an amount equivalent to the amplitude. For example, if the equation of the axis is $y = 7$ and the amplitude is 3, then the range would be $\{y \in \mathbf{R} \mid 4 \leq y \leq 10\}$.

Study Aid

- See Lesson 6.5, Example 2.
- Try Chapter Review Question 11.

Q: How do you determine the equation of a sinusoidal function from its graph?

A:

1. Use the formula

$$y = \frac{\text{maximum} + \text{minimum}}{2}$$

to determine the equation of the axis, which is equivalent to the vertical translation and the value of c.

2. Use the formula amplitude = maximum − axis to determine the amplitude of the function, which is equivalent to the vertical stretch or compression and the value of a. If the graph is reflected in the x-axis, then a is negative.

3. Use the formula

$$\text{period} = \frac{360°}{|k|}$$

to determine the horizontal stretch or compression, $\frac{1}{|k|}$.

4. Determine the horizontal translation. It is often easier to transform the function $y = \cos x$ than to transform $y = \sin x$ because, in many questions, it is easier to identify the coordinates of the peak of the function rather than points on the axis. If you are transforming $y = \cos x$, the horizontal translation is equivalent to the x-coordinate of any maximum. Determining this gives you the value of d.

5. Incorporate all the transformations into the equation $y = a\cos(k(x - d)) + c$ or $y = a\sin(k(x - d)) + c$.

Study Aid

- See Lesson 6.6, Example 1.
- Try Chapter Review Question 12.

PRACTICE Questions

Lesson 6.1

1. The automatic dishwasher in a school cafeteria runs constantly through lunch. The table shows the amount of water in the dishwasher at different times.

Time (min)	0	1	2	3	4	5	6	7
Volume (L)	0	16	16	16	16	16	0	16

Time (min)	8	9	10	11	12	13	14	15
Volume (L)	16	16	0	16	16	16	16	16

Time (min)	16	17	18	19	20
Volume (L)	0	16	16	16	0

a) Plot the data, and draw the resulting graph.
b) Is the graph periodic?
c) What is the period of the function, and what does it represent in this situation?
d) Determine the equation of the axis.
e) Determine the amplitude.
f) What is the range of this function?

2. Sketch a graph of a periodic function whose period is 20 and whose range is $\{y \in \mathbf{R} | 3 \leq y \leq 8\}$.

Lesson 6.2

3. Sketch the graph of a sinusoidal function that has a period of 6, an amplitude of 4, and whose equation of the axis is $y = -2$.

4. Colin is on a unique Ferris wheel: it is situated on the top of a building. Colin's height above the ground at various times is recorded in the table.

Time (s)	0	10	20	30	40	50
Height (m)	25	22.4	16	9.7	7	9.7

Time (s)	60	70	80	90	100	110
Height (m)	16	22.4	25	22.4	16	9.7

Time (s)	120	130	140	150	160
Height (m)	7	9.7	16	22.4	25

a) What is the period of the function, and what does it represent in this situation?
b) What is the equation of the axis, and what does it represent in this situation?
c) What is the amplitude of the function, and what does it represent in this situation?
d) Was the Ferris wheel already in motion when the data were recorded? Explain.
e) How fast is Colin travelling around the wheel, in metres per second?
f) What is the range of the function?
g) If the building is 6 m tall, what was Colin's boarding height in terms of the building?

Lesson 6.3

5. a) Graph the function $h(x) = 4\cos(3x) + 9$ using a graphing calculator in DEGREE mode for $0° \leq x \leq 360°$. Use Xscl $= 90°$. Determine the period, equation of the axis, amplitude, and the range of the function.
b) Is the function sinusoidal?
c) Calculate $h(45)$.
d) Determine the values of x, $0° \leq x \leq 360°$, for which $h(x) = 5$.

6. A ship is docked in port and rises and falls with the waves. The function $d(t) = 2\sin(30t)° + 5$ models the depth of the propeller, $d(t)$, in metres at t seconds. Graph the function using a graphing calculator, and answer the following questions.
a) What is the period of the function, and what does it represent in this situation?
b) If there were no waves, what would be the depth of the propeller?
c) What is the depth of the propeller at $t = 5.5$ s?
d) What is the range of the function?
e) Within the first 10 s, at what times is the propeller at a depth of 3 m?

7. Determine the coordinates of the image point after a rotation of 25° about (0, 0) from the point (4, 0).

Lesson 6.4

8. Each sinusoidal function has undergone one transformation that may have affected the period, amplitude, or equation of the axis of the function. In each case, determine which characteristic has been changed. If one has, indicate its new value.

a) $y = \sin x - 3$
b) $y = \sin(4x)$
c) $y = 7 \cos x$
d) $y = \cos(x - 70°)$

Lesson 6.5

9. Use transformations to graph each function for $0° \leq x \leq 360°$.
a) $y = 5 \cos(2x) + 7$
b) $y = -0.5 \sin(x - 30°) - 4$

10. Determine the range of each sinusoidal function without graphing.
a) $y = -3 \sin(4x) + 2$
b) $y = 0.5 \cos(3(x - 40°))$

Lesson 6.6

11. The average daily maximum temperature in Kenora, Ontario, is shown for each month.

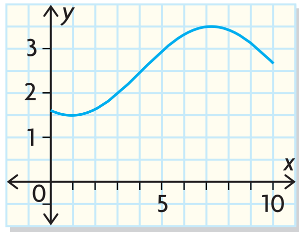
Time (months)	J	F	M	A
Temperature (°C)	−13.1	−9.0	−1.1	8.5

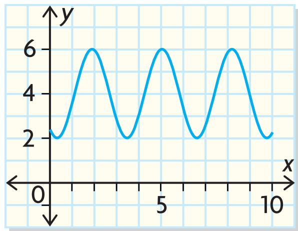
Time (months)	M	J	J	A
Temperature (°C)	16.8	21.6	24.7	22.9

Time (months)	S	O	N	D
Temperature (°C)	16.3	9.3	−1.2	−10.2

a) Prepare a scatter plot of the data. Let January represent month 0.
b) Draw a curve of good fit. Explain why this type of data can be expressed as a periodic function.
c) State the maximum and minimum values.
d) What is the period of the curve? Explain why this period is appropriate within the context of the question.
e) Write an equation for the axis of the curve.
f) What is the phase shift if the cosine function acts as the base curve?
g) Use the cosine function to write an equation that models the data.
h) Use the equation to predict the temperature for month 38. How can the table be used to confirm this prediction?

12. Determine the sine function $y = a \sin k(\theta - d) + c$ for each graph.

a)


b)


Lesson 6.7

13. Meagan is sitting in a rocking chair. The distance, $d(t)$, between the wall and the rear of the chair varies sinusoidally with time t. At $t = 1$ s, the chair is closest to the wall and $d(1) = 18$ cm. At $t = 1.75$ s, the chair is farthest from the wall and $d(1.75) = 34$ cm.
a) What is the period of the function, and what does it represent in this situation?
b) How far is the chair from the wall when no one is rocking in it?
c) If Meagan rocks back and forth 40 times only, what is the domain of the function?
d) What is the range of the function in part (c)?
e) What is the amplitude of the function, and what does it represent in this situation?
f) What is the equation of the sinusoidal function?
g) What is the distance between the wall and the chair at $t = 8$ s?

14. Summarize how you can determine the equation of a sinusoidal function that represents real phenomena from data, a graph, or a description of the situation. In your summary, explain how each part of the equation relates to the characteristics of the graph.