FREQUENTLY ASKED *Questions*

Q: **How can you use algebra to solve a linear system?**

A1: You can use a substitution strategy. Express one variable in one of the equations in terms of the other variable. Substitute this expression into the other equation, and solve for the remaining variable. Finally, substitute the solved value into the expression to determine the value of the other variable.

EXAMPLE

Solve the system.

$2x + y = 29$ ①

$4x - 3y = 18$ ②

Solution

From ①, $y = 29 - 2x$.

Substitute this expression for y into ② and solve for x.

$4x - 3(29 - 2x) = 18$

$4x - 87 + 6x = 18$

$10x - 87 = 18$

$10x = 18 + 87$

$10x = 105$

$x = 10.5$

Determine y by substituting this value of x into the expression for y.

$y = 29 - 2(10.5)$

$y = 8$

The solution is $x = 10.5$ and $y = 8$.

A2: You can use an elimination strategy. Eliminate x or y from the system by multiplying one or both equations by a constant other than zero, and then adding or subtracting the equations. Solve the resulting equation for the remaining variable. Substitute the solved value into one of the original equations, and determine the value of the other variable.

EXAMPLE

Solve the system.

$2x + y = 29$ ①

$4x - 3y = 18$ ②

Solution

Eliminating x	**Eliminating y**

Eliminating x

Multiply ① by 2 and subtract.

$$4x + 2y = 58 \qquad ① \times 2$$
$$\underline{4x - 3y = 18}$$
$$5y = 40$$
$$y = 8$$

Substitute $y = 8$ into ①.

$$2x + 8 = 29$$
$$2x = 29 - 8$$
$$2x = 21$$
$$x = 10.5$$

Eliminating y

Multiply ① by 3 and add.

$$6x + 3y = 87 \qquad ① \times 3$$
$$\underline{4x - 3y = 18}$$
$$10x = 105$$
$$x = 10.5$$

Substitute $x = 10.5$ into ①.

$$2(10.5) + y = 29$$
$$21 + y = 29$$
$$y = 29 - 21$$
$$y = 8$$

The solution is $x = 10.5$ and $y = 8$.

Q: **When you use elimination, how do you decide whether to add or subtract the equations?**

A: If the coefficients of the variable you want to eliminate are the same, subtract. If the coefficients are opposites, add.

Q: **How do you decide whether to use graphs, substitution, or elimination to solve a linear system?**

A: The strategy you use will depend on what degree of accuracy is required, what form the equations are written in, and whether more than just the solution is required. Algebraic solutions give exact answers, whereas hand-drawn graphs often do not.

Use graphs if
- you do not need an exact answer
- you need to look for trends or compare the graphs before and after the point of intersection
- you have a graphing calculator and both equations are in the form $y = mx + b$

Use substitution if
- you need exact answers
- one of the variables in the equation is already isolated, ready to make the substitution (that is, in the form $y = mx + b$)
- you can easily rearrange one equation to isolate a variable

Use elimination if
- you need exact answers
- both equations are in the form $Ax + By = C$ or $Ax + By + C = 0$

PRACTICE Questions

1. Sheila is planning to visit relatives in England and Spain. On the day that she wants to buy the currencies for her trip, one euro costs $1.50 and one British pound costs $2.00. What combinations of these currencies can Sheila buy for $700? Use three different strategies to show the possible combinations.

2. After a fundraiser, the treasurer for a minor soccer league invested some of the money in a savings account that paid 2.5%/year and the rest in a government bond that paid 3.5%/year. After one year, the money earned $140 in interest. Define two variables, write an equation, and draw a graph for this information.

3. Gary drove his pickup truck from Cornwall to Chatham. He left Cornwall at 8:15 a.m. and drove at a steady 100 km/h along Highway 401. The graph below shows how the fuel in the tank varied over time.

Amount of Fuel vs. Time
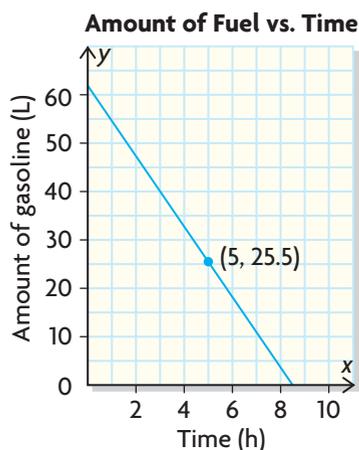
a) What do the coordinates of the point (5, 25.5) tell you about the amount of fuel?
b) How much fuel was in the tank at 11:45 a.m.?
c) The low fuel warning light came on when 6 L of fuel remained. At what time did this light come on?

4. Readycars charges $59/day plus $0.14/km to rent a car. Bestcars charges $69/day plus $0.11/km. Describe three different strategies you could use to compare these two rental rates. What advice would you give someone who wants to rent a car from one of these companies?

5. Solve each linear system graphically.
   a) $x + y = 2$
   $x = 2y + 2$
   b) $y - x = 1$
   $2x - y = 1$

6. Tools-R-Us rents snowblowers for a base fee of $20 plus $8/h. Randy's Rentals rents snowblowers for a base fee of $12 plus $10/h.
   a) Create an equation that represents the cost of renting a snowblower from Tools-R-Us.
   b) Create the corresponding equation for Randy's Rentals.
   c) Solve the system of equations graphically.
   d) What does the point of intersection mean in this situation?

7. Use substitution to solve each system.
   a) $2x + 3y = 7$
   $-2x - 1 = y$
   c) $5x + 2y = 18$
   $2x + 3y = 16$
   b) $3x - 4y = 5$
   $x - y = 5$
   d) $9 = 6x - 3y$
   $4x - 3y = 5$

8. Courtney paid a one-time registration fee to join a fitness club. She also pays a monthly fee. After three months, she had paid $315. After seven months, she had paid $535. Determine the registration fee and the monthly fee.

9. A rectangle has a perimeter of 40 m. Its length is 2 m greater than its width.
  a) Represent this situation with a linear system.
  b) Solve the linear system using substitution.
  c) What do the numbers in the solution represent? Explain.

Lesson 1.5

10. a) Which linear system below is equivalent to the system that is shown in the graph?
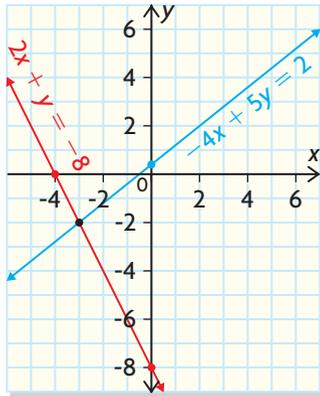
  A. $2x - 5y = 4$
    $-x + y = 1$

  B. $x - 3y = -1$
    $2x + y = 4$

  b) Use addition and subtraction to create another linear system that is equivalent to the system in the graph.
  c) Use multiplication to create another linear system that is equivalent to the system in the graph.

11. a) Create two linear systems that are equivalent to the following system.
    $-2x - 3y = 5$
    $3x - y = 9$
  b) Verify that all three systems have the same solution.

Lesson 1.6

12. Use elimination to solve each linear system.
  a) $2x - 3y = 13$
    $5x - y = 13$
  c) $3x + 21 = 5y$
    $4y + 6 = -9x$

  b) $x - 3y = 0$
    $3x - 2y = -7$
  d) $x - \dfrac{1}{3}y = -1$
    $\dfrac{2}{3}x - \dfrac{1}{4}y = -1$

13. Lyle needs 200 g of chocolate that is 86% cocoa for a cake recipe. He has one kind of chocolate that is 99% cocoa and another kind that is 70% cocoa. How much of each kind of chocolate does he need to make the cake? Round your answer to the nearest gram.

14. A Grade 10 class is raising money for a school-building project in Uganda. To buy 35 desks and 3 chalkboards, the students need to raise $2082. To buy 40 desks and 2 chalkboards, they need to raise $2238. Determine the cost of a desk and the cost of a chalkboard.

15. Solve the linear system.
    $2(2x - 1) - (y - 4) = 11$
    $3(1 - x) - 2(y - 3) = -7$

16. Juan is a cashier at a variety store. He has a total of $580 in bills. He has 76 bills, consisting of $5 bills and $10 bills. How many of each type does he have?

17. a) Sketch a linear system that has no solution.
  b) Determine two possible equations that could represent both lines in your sketch.
  c) Explain how the slopes of these lines are related.

18. The linear system $6x + 5y = 10$ and $ax + 2y = b$ has an infinite number of solutions. Determine a and b.