1.2 Solving Linear Equations

GOAL

Connect the solution to a linear equation and the graph of the corresponding relation.

YOU WILL NEED

- grid paper
- ruler
- graphing calculator

LEARN ABOUT the Math

Joe downloads music to his MP3 player from a site that charges $9.95 per month plus $0.55 for each song. Joe has budgeted $40 per month to spend on music downloads.

? How can Joe determine the greatest number of songs that he can download each month?

EXAMPLE 1 Selecting a strategy to solve the problem

Determine the maximum number of songs that Joe can download each month.

William's Solution: Solving a problem by reasoning

$\$40.00 - \$9.95 = \$30.05$ ← I calculated how much of Joe's budget he can spend on the songs he downloads, by subtracting the $9.95 monthly fee from $40.

$\$30.05 \div \$0.55 \doteq 54.63$ ← Each song costs $0.55, so I divided this into the amount he would have left to spend on songs.

Joe can download a maximum of 54 songs. ← I rounded down to 54, since 55 songs would cost more than he can spend.

Tony's Solution: Solving a problem by using an equation

Let n represent the number of songs and let C represent the cost.

$C = 9.95 + 0.55n$
$40 = 9.95 + 0.55n$ ← I created an equation and substituted the $40 Joe has budgeted for C.

$$40 - 9.95 = 9.95 + 0.55n - 9.95$$
$$30.05 = 0.55n$$
$$\frac{30.05}{0.55} = n$$
$$54.6 \doteq n$$

I solved for n using inverse operations.

Joe can download a maximum of 54 songs.

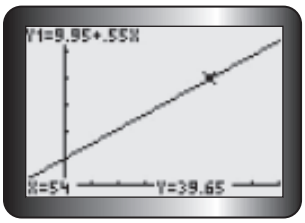
Since n has to be a whole number, I used the nearest whole number less than 54.6 for my answer.

Lucy's Solution: Solving a problem using graphing technology

Let X represent the number of songs and Y1 the cost.

$Y1 = 9.95 + 0.55X$

I entered the equation for the cost of music downloads into a graphing calculator. The number of songs downloaded has to be a whole number, so X represents a whole number. I graphed using Zoom Integer, so the x-values would go up by 1 when I traced the graph.



I used Trace to determine which point on the graph is closest to $y = 40$ (but less than \$40). This point is (54, 39.65).

Joe can download 54 songs in a month for \$39.65.

Tech Support

For help graphing and tracing along relations using a TI-83/84 graphing calculator, see Appendix B-2. If you are using a TI-*n*spire, see Appendix B-38.

Reflecting

A. How are William's and Tony's solutions similar? How are they different?

B. How did a single point on Lucy's graph represent a solution to the problem?

C. Which strategy do you prefer? Explain why.

APPLY the Math

EXAMPLE 2 Representing and solving a problem that involves a linear equation

At 9:20 a.m., Adrian left Windsor with 64 L of gas in his car. He drove east at 100 km/h. The low fuel warning light came on when 10 L of gas were left. Adrian's car uses gas at the rate of 8.8 L/100 km. When did the warning light come on?

Stefani's Solution: Solving an equation algebraically

Adrian's car uses 8.8 L of gas every 100 km. Since he drove at 100 km/h, he used 8.8 L/h.

I calculated how much gas the car used each hour.

$G = 64 - 8.8t$

I wrote an equation for the amount of gas used. I let t represent the time in hours, and I let G represent the amount of gas in litres.

$$\begin{aligned} 10 &= 64 - 8.8t \\ 10 - 10 &= 64 - 8.8t - 10 \\ 0 &= 54 - 8.8t \\ 8.8t &= 54 \\ t &= \frac{54}{8.8} \\ t &\doteq 6.14 \end{aligned}$$

The warning light came on when $G = 10$, so I let $G = 10$ and solved for t using inverse operations.

The warning light came on after Adrian had been driving about 6.14 h.

$0.14 \times 60 = 8.4$

I wrote the time in hours and minutes by multiplying the part of the number to the right of the decimal point by 60.

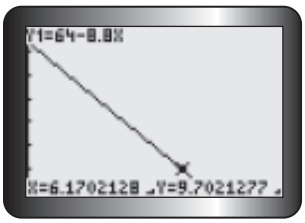
The warning light came on about 6 h 8 min after 9:20 a.m., which is about 3:28 p.m.

Henri's Solution: Solving a problem by using a graph

$y = 64 - 8.8x$

I wrote an equation for the amount of gas in the tank at any time. I let x represent the time in hours, and I let y represent the amount of gas in litres.

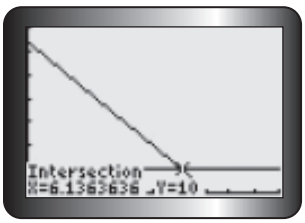
Graph Y1 = 64 − 8.8X.



After about 6.17 h, there was about 9.7 L of gas in the tank.

I graphed the equation on a graphing calculator. I knew that the *y*-intercept was 64, and I estimated that the *x*-intercept was about 7, so I used the window settings shown.

I used Trace to locate the point with a *y*-value closest to 10.

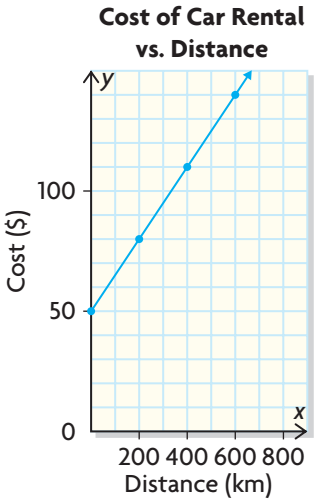


To get an exact solution, I entered the line Y2 = 10. The *x*-coordinate of the **point of intersection** between the two lines tells the time when 10 L of gas is left in the tank.

Based on the graph, the warning light came on about 6.14 h after Adrian started, at about 3:28 p.m.

Tech *Support*

For help determining the point of intersection between two relations on a TI-83/84 graphing calculator, see Appendix B-11. If you are using a TI-*n*spire, see Appendix B-47.



In Summary

Key Idea

- You can solve a problem that involves a linear relation by solving the associated linear equation.

Need to Know

- You can solve a linear equation in one variable by graphing the associated linear relation and using the appropriate coordinate of an ordered pair on the line. For example, to solve $3x - 2 = 89$, graph $y = 3x - 2$ and look for the value of x at the point where $y = 89$ on the line.

CHECK Your Understanding

1. Estimate solutions to the following questions using the graph at the left.

a) What is the rental cost to drive 500 km?

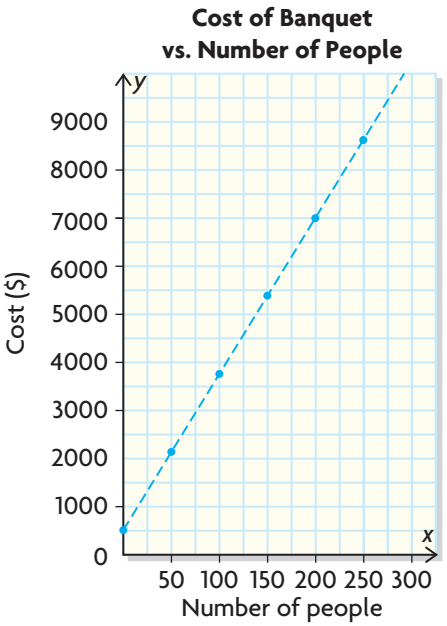
b) How far can you drive for $80, $100, and $75?

2. **a)** Write an equation for the linear relation in question 1.
b) Use your equation to answer question 1.
c) Compare your answers for question 1 with your answers for part b) above. Which strategy gave the more accurate answers?

3. Apple juice is leaking from a carton at the rate of 5 mL/min. There are 1890 mL of juice in the container at 10:00 a.m.
a) Write an equation for this situation, and draw a graph.
b) When will 1 L of juice be left in the carton?

PRACTISING

4. K The graph at the right shows how the charge for a banquet hall relates to the number of people attending a banquet.
a) Locate the point (160, 5700) on the graph. What do these coordinates tell you about the charge for the banquet hall?
b) What is the charge for the banquet hall if 200 people attend?
c) Write an equation for this linear relation.
d) Use your equation to determine how many people can attend for $3100, $4400, and $5000.
e) Why is a broken line used for this graph?



5. Max read on the Internet that 1 U.S. gallon is approximately equal to 3.785 L.
a) Draw a graph that you can use to convert U.S. gallons into litres.
b) Use your graph to estimate the number of litres in 6 gallons.
c) Use your graph to estimate the number of gallons in 14 L.

6. Melanie drove at 100 km/h from Ajax to Ottawa. She left Ajax at 2:15 p.m., with 35 L of gas in the tank. The low fuel warning light came on when 9 L was left in the tank. If Melanie's SUV uses gas at the rate of 9.5 L/100 km, estimate when the warning light came on.

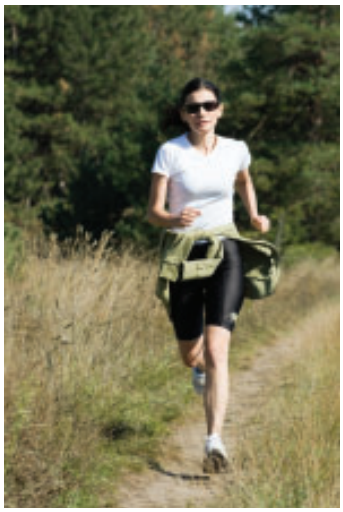
7. Hank sells furniture and earns $280/week plus 4% commission.
a) Determine the sales that Hank needs to make to meet his weekly budget requirement of $900.
b) Write an equation for this situation, and use it to verify your answer for part a).

8. A The Perfect Paving Company charges $10 per square foot to install interlocking paving stones, as well as a $40 delivery fee.
a) Determine the greatest area that Andrew can pave for $3500.
b) Andrew needs to include 5 cubic yards of sand, costing $15 per cubic yard, to the total cost of the project. How much will this added cost reduce the area that he can pave with his $3500 budget?

9. A student athletic council raised $4000 for new sports equipment and uniforms, which will be purchased 3 years from now. Until then, the money will be invested in a simple interest savings account that pays 3.5%/year.
   a) Write an equation and draw a graph to represent the relationship between time (in years) and the total value of their investment.
   b) Use the graph to determine the value of their investment after 2 years.
   c) Use the equation to determine when their investment is worth $4385.

10. Maria has budgeted $90 to take her grandmother for a drive. Katey's Kars rents cars for $65 per day plus $0.12/km. Determine how far Maria and her grandmother can travel, including the return trip.

11. C Cam earns $400/week plus 2.5% commission. He has been offered another job that pays $700/week but no commission.
   a) Describe three strategies that you could use to compare Cam's earnings for the two jobs.
   b) Which job should Cam take? Justify your decision.



Health *Connection*

Jogging is an exercise that keeps you healthy and can burn about 650 calories per hour.

12. T At 9:00 a.m., Chantelle starts jogging north at 6 km/h from the south end of a 21 km trail. At the same time, Amit begins cycling south at 15 km/h from the north end of the same trail. Use a graph to determine when they will meet.

13. Explain how to determine the value of x, both graphically and algebraically, in the linear relation $2x - 3y = 6$ when $y = 5$.

Extending

14. The owner of a dart-throwing stand at a carnival pays 75¢ every time the bull's-eye is hit, but charges 25¢ every time it is missed. After 25 tries, Luke paid $5.25. How many times did he hit the bull's-eye?

15. Adriana earns 5% commission on her sales up to $25 000, 5.5% on any sales between $25 000 and $35 000, 6% on any sales between $35 000 and $45 000, and 7% for any sales over $45 000. Draw a graph to represent how Adriana's earnings depend on her sales. What sales volume does she need to earn $2000?

Number of Buttons	Cost per Button ($)
1 to 25	1.00
26 to 50	0.80
51 to 100	0.60
101 or more	0.20

16. A fabric store sells fancy buttons for the prices in the table at the left.
   a) Make a table of values and draw a graph to show the cost of 0 to 125 buttons.
   b) Compare the cost of 100 buttons with the cost of 101 buttons. What advice would you give someone who needed 100 buttons? Comment on this pricing structure.
   c) Write equations to describe the relationship between the cost and the number of buttons purchased.