(A) Applications

Example #1: I invest $45,000 in a funding yielding 6% p.a. compounded continuously.

(a) Find the value of the investment after 5 years.
(b) How long does it take for the investment to triple in value?

Example #2: A certain bacteria grows according to the formula $A(t) = 5000e^{0.4055t}$, where t is time in hours.

(a) What will the population be in 8 hours
(b) When will the population reach 1,000,000

Example #3: The population of the USA can be modeled by the eqn $P(t) = 227e^{0.0093t}$, where P is population in millions and t is time in years since 1980

(a) What is the annual growth rate?
(b) What is the predicted population in 2015?
(c) What assumptions are being made in question (b)?
(d) When will the population reach 500 million?

Example #4: The function $P(t) = 1 - e^{-0.0479t}$ gives the percentage of the population that has seen a new TV show *t* weeks after it goes on the air.

(a) What percentage of people have seen the show after 24 weeks?
(b) Approximately, when will 90% of the people have seen the show?
(c) What happens to P(t) as t gets infinitely large? Why? Is this reasonable?

Example #5: The number of bacteria in a culture is given by the function $n(t) = 10e^{0.22t}$

(a) What is the relative rate of growth of this bacterium population? Express your answer as a percentage.
(b) What is the initial population of the culture (at t = 0)?
(c) How many bacteria will be in the culture at time t = 15?
(d) What is the doubling time for this bacterial population?

Example #6: A colony of bacteria exhibits continuous growth. The initial population in a sample of this bacteria is 36 and there are 72 bacteria after one hour.

(a) Determine the number of bacteria after 8 hours.
(b) Determine an exponential model for *N*, the number of bacteria after *t* hours i.e. N(t) = ???
(c) Determine the growth rate PER DAY.

Example #7: The population of a town is continuously changing. The population of a small town appears to be increasing exponentially. Town planners need a model for predicting the future population. In the year 2000, the population was 35,000, while in the year 2010, the population grew to 57,010.

(a) PREDICT: What will be the town's population in 2030?
(b) Create an **exponential** algebraic model for the town's population growth.
(c) Check your population model by using the fact that the town's population was 72,825 in 2015.
(d) CALCULATE: What will be the town's population in 2030?

Example #8: Three years ago, the fish population in Loon Lake was 2500. Due to the effects of acid rain, there are now about 1950 fish in the lake. Assume that the decline of the fish population is exponential and happens continuously. Find the predicted fish population 5 years from now.

Example #9: What is the average annual rate of inflation if a loaf of bread cost $1.19 in 1991 but costs $1.50 in 2001

HL EXTENION: Summations: Further Investigations

Consider the following number pattern:

$$\frac{1}{1}, \frac{1}{1\times 2}, \frac{1}{1\times 2\times 3}, \frac{1}{1\times 2\times 3\times 4}, \frac{1}{1\times 2\times 3\times 4\times 5}, \ldots..$$

(a) Describe what is happening in this sequence of terms
(b) Predict the next 3 terms
(c) Determine the sum of the first 5 terms: $\frac{1}{1} + \frac{1}{1\times 2} + \frac{1}{1\times 2\times 3} + \frac{1}{1\times 2\times 3\times 4} + \frac{1}{1\times 2\times 3\times 4\times 5}$
(d) Determine the sum of the first 8 terms
(e) This sum of terms in the number pattern can be mathematically expressed using the summation formula $\sum_{x=0}^{n}\left(\frac{1}{x!}\right)$. Generate some more terms and sums and see what happens as we progress further into the series.