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Name: ______________________ Date : ____________

IM 3 UNIT TEST V4 - Quadratic Functions

Teacher: Mr. Santowski and Ms. Aschenbrenner

Score:______

PART 2 - CALCULATOR INACTIVE QUESTIONS

.Show all work and write all answers in the spaces provided. Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided the answer is supported by written work.

1. Solve each of the following equations using the ALGEBRAIC method of your choosing. **Show the key steps of your algebraic working in order to earn FULL CREDIT for correct answers.**

(6 marks)

(f) $0 = 6(x-7)^2 - 96$

(3M)

$\frac{96}{6} = \frac{6}{6}(x-7)^2$

$16 = (x-7)^2$

$\pm 4 = (x-7)$

$7 \pm 4 = x$

$\boxed{11, 3 = x}$

(g) $7x + 8 = 6x^2 + 5$

(3M)

$0 = 6x^2 - 7x - 3$ -18, -7, $-9, 2$

$0 = 6x^2 - 9x + 2x - 3$

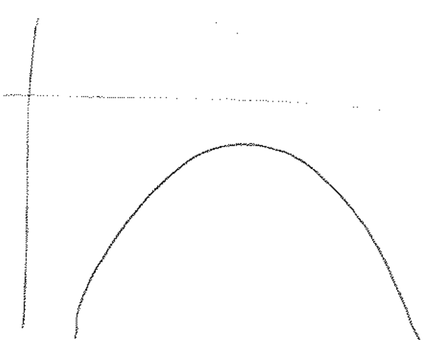
$0 = 3x(2x-3) + 1(2x-3)$

$0 = (3x+1)(2x-3)$

$\boxed{x = -\frac{1}{3}, \frac{3}{2}}$

2. Write the EQUATION of a parabola that has the following characteristics: (i) opens downward **AND** (ii) has no zeroes **AND** (iii) is wider than the parent function, $y = x^2$ **AND** (iv) has been translated to the right, compared to its parent function of $y = x^2$

(4 marks)



$y = -\frac{1}{2}(x-4)^2 - 3$

3. For each of the quadratic functions listed below, determine the optimal VALUE of each.

(6 marks)

(a) $g(x) = 3x^2 + 12x - 9$ using completing the square

$g(x) + 9 = 3(x^2 + 4x)$

$g(x) + 9 + 12 = 3(x^2 + 4x + 4)$

$g(x) + 21 = 3(x+2)^2$

$g(x) = 3(x+2)^2 - 21$

$\boxed{(-2, -21)}$

(b) $f(x) = -4x(x + 8) - 6$ using the method of your choosing

$f(x) = -4x^2 - 32x - 6$

$x = +\frac{+32}{2(-4)} = \frac{32}{-8} = -4$

$f(-4) = +4(+4)(-4+8) - 6$

$= 16(4) - 6$

$= 64 - 6$

$= 58$

$\boxed{(-4, 58)}$

4. In this question, you will work with the quadratic function $f(x) = -\frac{1}{4}(x+2)(x-6)$.

(14 marks)

(h) Where are the zeroes of this function?

(1)

$(-2, 0)$

$(6, 0)$

(i) Where is the y-intercept of this function?

$f(0) = +\frac{1}{\cancel{4}}(2)(\cancel{+6})$ 3

$f(0) = 3$ $\boxed{(0, 3)}$ (1)

(a) Write the equation in vertex form.

(3) $x = \frac{-2+6}{2} = 2$

$f(2) = +\cancel{\frac{1}{4}(4)}(+4)$

$= 4$

$\boxed{f(x) = -\frac{1}{4}(x-2)^2 + 4}$

(b) Write the equation in standard form.

$f(x) = -\frac{1}{4}(x^2 - 4x - 12)$ (1)

$\boxed{f(x) = -\frac{1}{4}x^2 + x + 3}$

4. CONTINUED: In this question you will work with the quadratic function $f(x) = -\frac{1}{4}(x+2)(x-6)$.

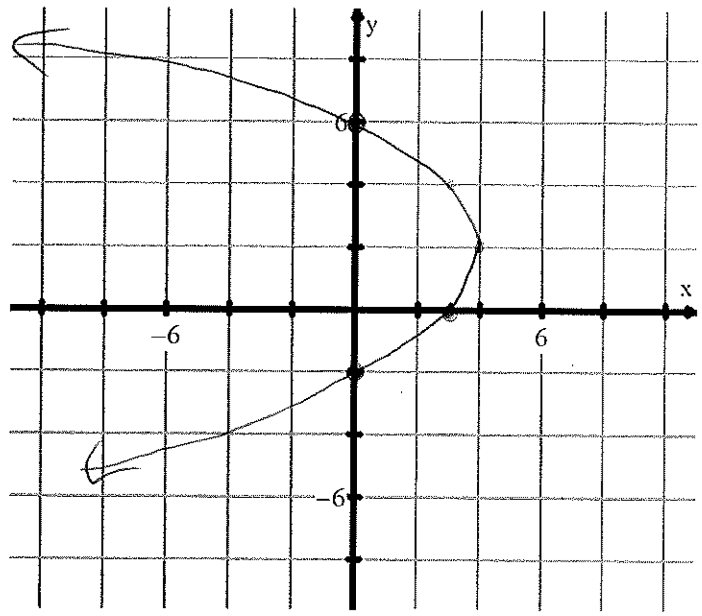
(c) This function represents a transformed parabola. List the transformations that were applied to the parent function, $y = x^2$.

(2)

- reflect across x-axis
- stretch vertically by a factor of $\frac{1}{4}$
- shift right 2
- shift up 4

(d) Graph the INVERSE RELATION (including the exact location of its vertex and intercepts).

(3)



(e) The graph of the inverse relation you just drew is NOT a function. By making certain domain/range restrictions, we can make this RELATION into a function. Explain one restriction that can be made to create a FUNCTION from this RELATION.

(1)

restrict range of inverse relation to $y \geq 2$ or $y \leq 2$

restrict domain of original function to $x \geq 2$ or $x \leq 2$

(f) Find the equation of the inverse function.

(2)

$f(x) = -\frac{1}{4}(x-2)^2+4$

$x = -\frac{1}{4}(y-2)^2+4$

$-4 \quad (x-4) = -\frac{1}{4}(y-2)^2 \quad \cdot -4$

$-4x+16 = (y-2)^2$

$\pm\sqrt{-4x+16} = y-2$

$f^{-1}(x) = 2 \pm \sqrt{-4x+16}$

$f^{-1}(x) = 2 + \sqrt{-4x+16}$

or

$2 - \sqrt{-4x+16}$

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PART 1 - CALCULATOR ACTIVE QUESTIONS

Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided the answer is supported by working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer

1. In this question, you will be GRAPHICALLY solving the inequality $3t^2 + 9t - 5 > -t^2 + 4t + 3$

(6 marks)

(a) State the solution.

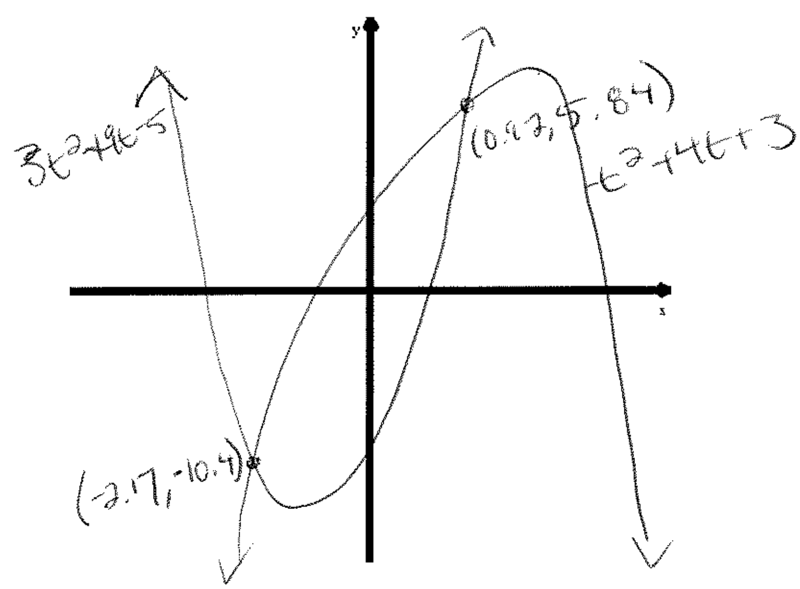
(2) $t < -2.17$ or $t > 0.92$

(b) What does the solution mean? (1)

~~The t~~ values where $t < -2.17$ and when $t > 0.92$ will make the y values of $3t^2+9t-5$ greater than the y values of $-t^2+4t+3$

(c) Draw a sketch of the two curves, labelling the curves and the intersections. (NOTE: You may use your TI-84 to help with this)

(3)



2. Here is an equation that models the PROFIT of a company that sells teddy bears: $P(n) = -0.1n^2 + 30n - 1200$, where P represents the company's weekly profit, in dollars and where n is the number of teddy bears sold in a week. **Show the key steps of your algebraic working in order to earn FULL CREDIT for correct answers.**

(12 marks)

(a) Evaluate P(400) and interpret.

(2)

$P(400) = -0.1(400)^2 + 30(400) - 1200$

$=$

When the company sell 400 teddy bears they will have a profit of $-5200

(b) How many teddy bears must the company sell to make $700?

$700 = -0.1n^2 + 30n - 1200$ (3)

90.8 or 209.2

91 or 209 teddy bears

(c) How many teddy bears must the company sell to maximize their profits?

(2)

150

(d) What is the maximum profit of the business?

(1)

$1050

(e) How many teddy bears must the company sell to break even?

$0 = P(n)$

(2)

$n = 47.53, 252.47$

47 or 252 teddy bears

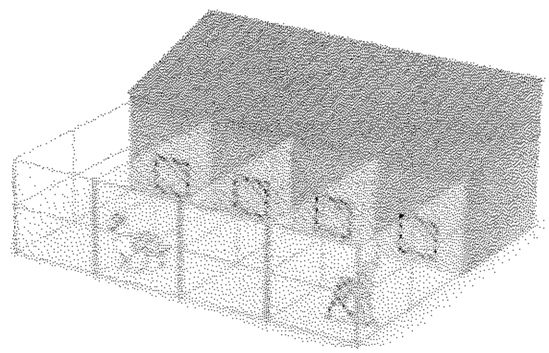
(f) The company has a goal to make **at least** $1000 by selling 170 teddy bears. **According to the mathematical model**, is this an attainable goal? Why or why not?

$P(170) = 1010$

(2)

Yes an attainable goal, according to the model the profit will be $1010.

3. A dog owner is building 4 individual kennels by fencing along the side of a building (below is a picture of what how the fence will be arranged, given the location of the fencing relative to a building.) She only has 250 m of fencing available to build these kennels for the dogs. (7 marks)



(a) EXPLAIN WHY the equation for the perimeter of the fence is $P = L + 5W$

(1) The fence is created with one long length and 5 widths the same size.

(b) SHOW HOW the equation for the AREA inside the fence can be written as $A = L\left(\frac{250 - L}{5}\right)$ (2)

$250 = L + 5W$

$250 - L = 5W$

$\frac{250-L}{5} = W$

$A = L \cdot W$

$A = L\left(\frac{250-L}{5}\right)$

(c) Using these 250m of fencing, what is the maximum area that can be created for the dogs? Show the **ALGEBRAIC analysis** that leads to your solution.

(3)

$A = L\left(\frac{250-L}{5}\right)$

$L = 0$ $\quad \frac{250-L}{5} = 0$

$250 - L = 0$

$L = 250$

$x = \frac{0+250}{2} = \frac{250}{2} = 125$

$A = 125\left(\frac{250-125}{5}\right)$

$= \boxed{3125 \text{ m}^2}$

(d) Finally, what are the lengths and widths of each of the kennels, given your answer in Q3(c) ? (1)

125m by 25m

4. Mr. S. has been researching a computer software company, as he has made significant financial investments in this company. He knows that their annual revenues (sales) are modeled by the function $R(t) = 4t^2 + 2t + 980$. The company's annual expenses (costs) are modeled by the function $E(t) = 900 + 50t$ (where $t = 0$ is the years since the study began in 2000).

(7 marks)

(a) Explain how we know ***from the equation*** that the company's expenses are increasing. (1)

The slope is positive which means the graph is increasing

(b) The axis of symmetry of the quadratic function modeling the revenues is t = - ¼ . Therefore, explain how we know ***from the equation*** that the revenues are increasing as well. (1)

The vertex is a negative and the a-value is positive therefore since t starts at 0, after 0, the parabola will be increasing.

(c) What were the company's revenues and expenses at the beginning of the study (in the year 2000?) (2)

$R(0) = 980$

$E(0) = 900$

(d) When does the company break even in its **profits**? (2)

$P = R(t) - E(t)$

$t = 2 + 10$

2002 + 2010

(e) During which years do the revenues **exceed** the expenses of the company? (1)

$R(t) > E(t)$

$\boxed{0 \leq t < 2 \text{ or } t > 10}$