CAC
CAIRO AMERICAN

Name: _____ Date : _____

IM 3 Mid Unit Quiz V2 - Functions
Teacher: Ms. Aschenbrenner, Mr. Santowski

Score: 37/~~40~~

CALCULATOR ACTIVE - SHOW RELEVANT WORK AND WRITE ALL ANSWERS IN THE SPACES PROVIDED.

A graphing display calculator MAY be used in this quiz. Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. The given relation, $y = f(x)$, is defined by the following graph

(9 marks)

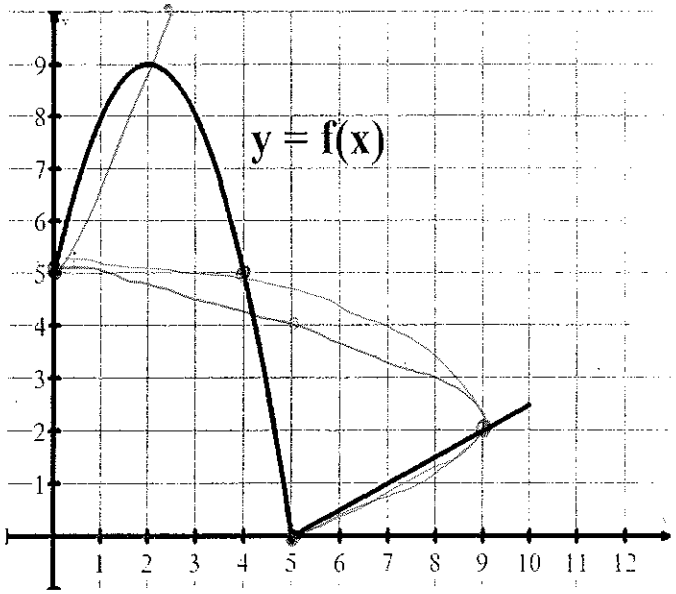
a) Evaluate $f(3)$ $= 8$ ✓

b) Solve $5 = f(x)$

$x = 0, 4$ ✓

c) Evaluate $f^{-1}(0)$ $= 5$ ✓

$f(x) = 0$



d) On what interval of the domain is $y = f(x)$ increasing?

$0 < x < 2$ or $5 < x < 10$

✓ ✓

e) List the optimal point(s) of $y = f(x)$

$(2, 9)$ $(5, 0)$

✓ ✓

f) Would the graph of the INVERSE of $y = f(x)$ be a function? Why or why not?

✓ no the y-value of 6 on f(x) has two x-values ✓ therefore of $f^{-1}(x)$ the x-value of 6 will have two y-values.

9

2. You are given the linear function $g(x) = 2x + 3$.

(9 marks)

a. List two points that would be on the graph of $g^{-1}(x)$. Show/explain the key steps of your answer.

ex) $g(1) = 5$
$g(0) = 3$
points on $g(x)$

therefore

points on $g^{-1}(x)$
$(5, 1)$ ✓
$(3, 0)$ ✓

b. Evaluate $g^{-1}(7)$. Show/explain the key steps of your answer.

$7 = 2x + 3$ ✓
$4 = 2x$
$2 = x$ ✓
$\therefore g^{-1}(7) = 2$

c. Solve $g^{-1}(x) = 1$. Show/explain the key steps of your answer.

$2(1) + 3 = g(x)$ ✓
$5 = g(x)$
$\therefore g^{-1}(x) = 1$
$x = 5$ ✓

d. Determine the equation of the inverse and write the equation as $g^{-1}(x) = Ax + B$.

$x = 2y + 3$ ✓
$x - 3 = 2y$
$\frac{x-3}{2} = g^{-1}(x)$
$g^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$ ✓

e. Solve the equation $g(x) = x + g^{-1}(x)$.

(1) Show/explain the key steps of your answer.

$2x + 3 = x + \frac{1}{2}x - \frac{3}{2}$ ✓
$\frac{4}{2}x + 3 = \frac{3}{2}x - \frac{3}{2}$
$-\frac{3}{2}x \quad -3 \quad -3\frac{6}{2}$
$\frac{1}{2}x = -\frac{9}{2}$
$x = -9$ ✓

9

3. The monthly sales for a new restaurant in Maadi is modeled by the equation $S(m) = -\frac{1}{2}m^3 + 6m^2 - 16m + 16$, where S is the monthly sales in thousands of dollars and m is the number of months since the restaurant first opened.

10 (12 marks)

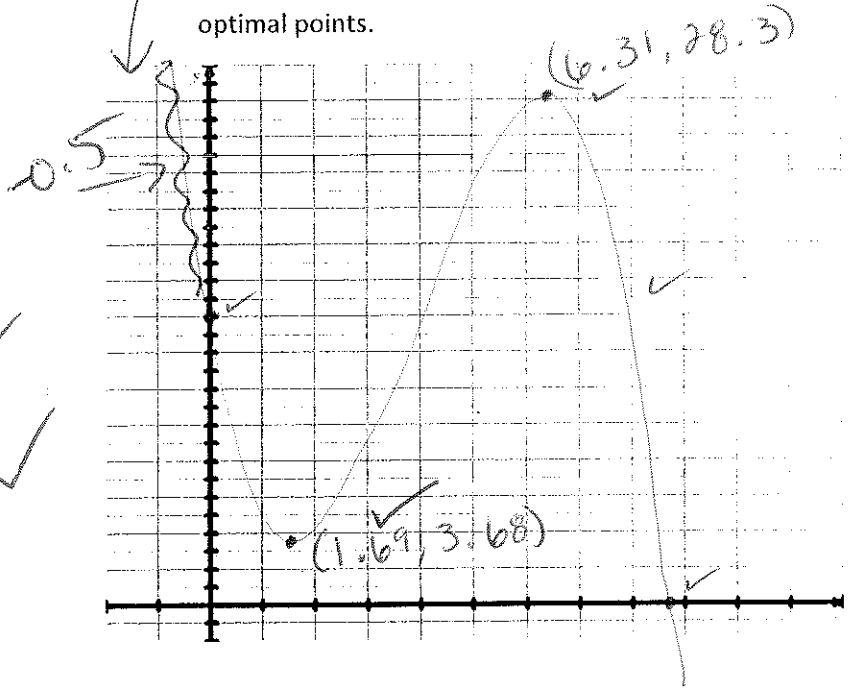
a) Graph the function on the TI-84 using these window settings:

Xmin = -5 Xmax = 15

Ymin = -5 Ymax = 40

min/max ✓
shape ✓
intercepts ✓

b) Sketch the function on the grid provided. Label the optimal points.



c) When are sales decreasing?

$0 < m < 1.69$ ✓
and
$6.31 < m < 8.77$ ✓

d) Determine the co-ordinates of the maximum point and explain what the point means in the context of this problem.

(6.31, 28.3) ✓
In June, the ~~profit~~ sales reach a maximum of 28.3 thousand dollars. ✓

e) The domain of the model is from 0 to M, where M is the last month before the function values become negative (as thus the equation is now useful for modeling the monthly sales.) Find this value of M and thus state the domain and range of S(m)

$M = 8.77$ ✓

D: $\{m \in \mathbb{R} \mid 0 \le m \le 8.77\}$ ✓

R: $\{y \in \mathbb{R} \mid 0 \le y \le 28.33\}$

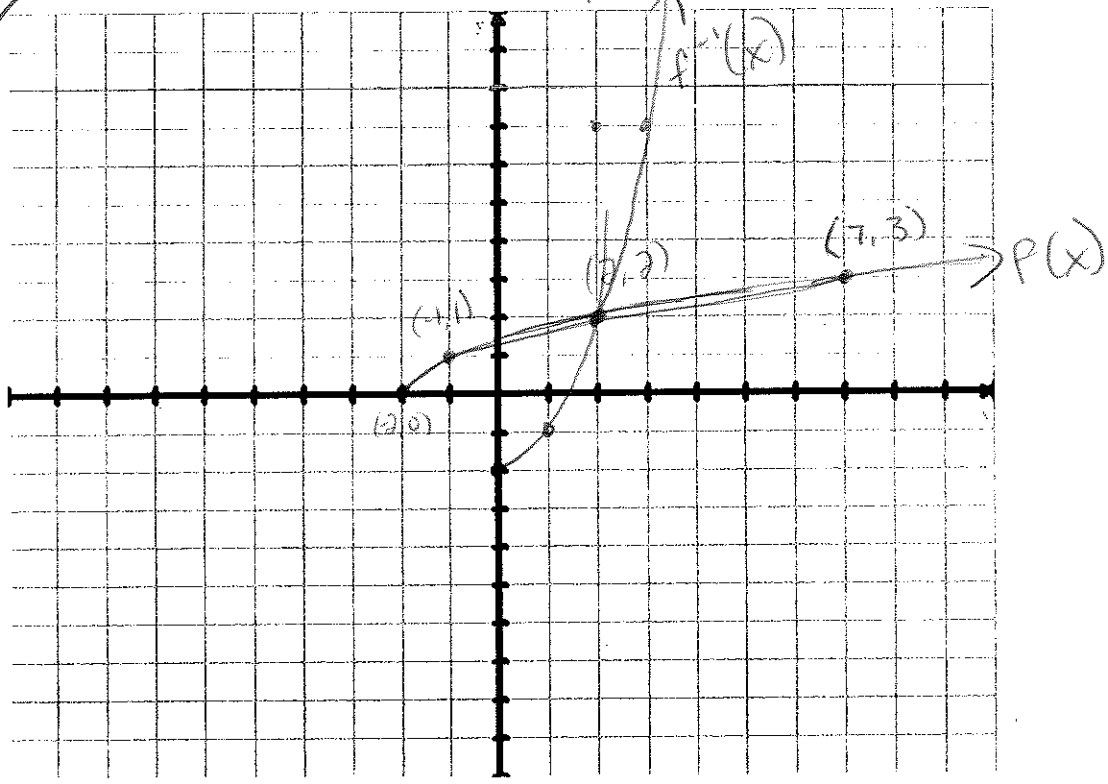
10

4. You will work with the function $f(x) = \sqrt{x+2}$. Use your calculator to graph the function in a standard view window.

(10 marks)

a) Sketch the function on the grid provided below. Label at least two points on the function.

shape ✓ points ✓



b) State the domain and range of the inverse

D: {x ∈ R | x ≥ 0} ✓

R: {y ∈ R | y ≥ -2} ✓

c) List two points on $f^{-1}(x)$.

(0, -2) (1, -1)

✓ ✓

d) Sketch the inverse of $f(x) = \sqrt{x+2}$ on the same grid.

shape ✓ points ✓

e) Determine the equation of the inverse. Show the key steps of your work. Write your final answer in function form.

$x = \sqrt{y+2}$ ✓

$x^2 - 2 = y$

$f^{-1}(x) = x^2 - 2, \quad x \geq 0$ ✓

10